

MATHEMATICAL GEOGRAPHY

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JOHNSON MATH. GEO.

PREFACE

IN the greatly awakened interest in the common-school subjects during recent years, geography has received a large share. The establishment of chairs of geography in some of our greatest universities, the giving of college courses in physiography, meteorology, and commerce, and the general extension of geography courses in normal schools, academies, and high schools, may be cited as evidence of this growing appreciation of the importance of the subject.

While physiographic processes and resulting land forms occupy a large place in geographical control, the earth in its simple mathematical aspects should be better understood than it generally is, and mathematical geography deserves a larger place in the literature of the subject than the few pages generally given to it in our physical geographies and elementary astronomies. It is generally conceded that the mathematical portion of geography is the most difficult, the most poorly taught and least understood, and that students require the most help in understanding it. The subject-matter of mathematical geography is scattered about in many works, and no one book treats the subject with any degree of thoroughness, or even makes a pretense at doing so. It is with the view of meeting the need for such a volume that this work has been undertaken.

Although designed for use in secondary schools and for teachers' preparation, much material herein organized may be used

in the upper grades of the elementary school. The subject has not been presented from the point of view of a little child, but an attempt has been made to keep its scope within the attainments of a student in a normal school, academy, or high school. If a very short course in mathematical geography is given, or if students are relatively advanced, much of the subject-matter may be omitted or given as special reports.

To the student or teacher who finds some portions too difficult, it is suggested that the discussions which seem obscure at first reading are often made clear by additional explanation given farther on in the book. Usually the second study of a topic which seems too difficult should be deferred until the entire chapter has been read over carefully.

The experimental work which is suggested is given for the purpose of making the principles studied concrete and vivid. The measure of the educational value of a laboratory exercise in a school of secondary grade is not found in the academic results obtained, but in the attainment of a conception of a process. The student's determination of latitude, for example, may not be of much value if its worth is estimated in terms of facts obtained, but the forming of the conception of the process is a result of inestimable educational value. Much time may be wasted, however, if the student is required to rediscover the facts and laws of nature which are often so simple that to see is to accept and understand.

Acknowledgments are due to many eminent scholars for suggestions, verification of data, and other valuable assistance in the preparation of this book.

To President George W. Nash of the Northern Normal and Industrial School, who carefully read the entire manuscript and proof, and to whose thorough training, clear insight, and kindly interest the author is under deep obligations, especial credit

is gratefully accorded. While the author has not availed himself of the direct assistance of his sometime teacher, Professor Frank E. Mitchell, now head of the department of Geography and Geology of the State Normal School at Oshkosh, Wisconsin, he wishes formally to acknowledge his obligation to him for an abiding interest in the subject. For the critical examination of portions of the manuscript bearing upon fields in which they are acknowledged authorities, grateful acknowledgment is extended to Professor Francis P. Leavenworth, head of the department of Astronomy of the University of Minnesota; to Lieutenant-Commander E. E. Hayden, head of the department of Chronometers and Time Service of the United States Naval Observatory, Washington; to President F. W. McNair of the Michigan College of Mines; to Professor Cleveland Abbe of the United States Weather Bureau; to President Robert S. Woodward of the Carnegie Institution of Washington; to Professor T. C. Chamberlin, head of the department of Geology of the University of Chicago; and to Professor Charles R. Dryer, head of the department of Geography of the State Normal School at Terre Haute, Indiana. For any errors or defects in the book, the author alone is responsible.

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CHAPTER I

INTRODUCTORY

OBSERVATIONS AND EXPERIMENTS

Observations of the Stars. On the first clear evening, observe the “Big Dipper”* and the polestar. In September and in December, early in the evening, they will be nearly in the positions represented in Figure 1. Where is the Big Dipper later in the evening? Find out by observations.

Learn readily to pick out Cassiopeia’s Chair and the Little Dipper. Observe their apparent motions also. Notice the positions of stars in different portions of the sky and observe where they are later in the evening. Do the stars around the polestar remain in the same position in relation

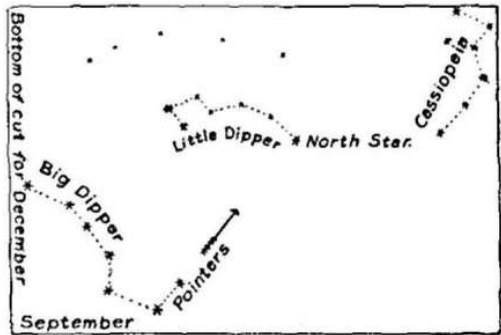


Fig. 1

to each other,—the Big Dipper always like a dipper, Cassiopeia’s Chair always like a chair, and both always on opposite sides of

*In Ursa Major, commonly called the “Plow,” “The Great Wagon,” or “Charles’s Wagon” in England, Norway, Germany, and other countries.

the polestar? In what sense may they be called “fixed” stars (see pp. 109, 265)?

Make a sketch of the Big Dipper and the polestar, recording the date and time of observation. Preserve your sketch for future reference, marking it Exhibit 1. A month or so later, sketch again at the same time of night, using the same sheet of paper with a common polestar for both sketches. In making your sketches be careful to get the angle formed by a line through the “pointers” and the polestar with a perpendicular to the horizon. This angle can be formed by observing the side of a building and the pointer line. It can be measured more accurately in the fall months with a pair of dividers having straight edges, by placing one outer edge next to the perpendicular side of a north window and opening the dividers until the other outside edge is parallel to the pointer line (see Fig. 2). Now lay the dividers on a sheet of paper and mark the angle thus formed, representing the positions of stars with asterisks. Two penny rulers pinned through the ends will serve for a pair of dividers.

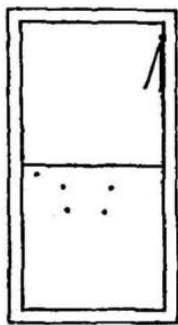


Fig. 2

Phases of the Moon. Note the position of the moon in the sky on successive nights at the same hour. Where does the moon rise? Does it rise at the same time from day to day? When the full moon may be observed at sunset, where is it? At sunrise? When there is a full moon at midnight, where is it? Assume it is sunset and the moon is high in the sky, how much of the lighted part can be seen?

Answers to the foregoing questions should be based upon first-hand observations. If the questions cannot easily be answered, begin observations at the first opportunity. Perhaps the best time to begin is when both sun and moon may be seen above the horizon. At each observation

notice the position of the sun and of the moon, the portion of the lighted part which is turned toward the earth, and bear in mind the simple fact that *the moon always shows a lighted half to the sun*. If the moon is rising when the sun is setting, or the sun is rising when the moon is setting, the observer must be standing directly between them, or approximately so. With the sun at your back in the east and facing the moon in the west, you see the moon as though you were at the sun. How much of the lighted part of the moon is then seen? By far the best apparatus for illustrating the phases of the moon is the sun and moon themselves, especially when both are observed above the horizon.

The Noon Shadow. Some time early in the term from a convenient south window, measure upon the floor the length of the shadow when it is shortest during the day. Record the measurement and the date and time of day. Repeat the measurement each week. Mark this Exhibit 2.

On a south-facing window sill, strike a north-south line (methods for doing this are discussed on pp. 59, 130). Erect at the south end of this line a perpendicular board, say six inches wide and two feet long, with the edge next the north-south line. True it with a plumb line; one made with a bullet and a thread will do. This should be so placed that the shadow from the edge of the board may be recorded on the window sill from 11 o'clock, A.M., until

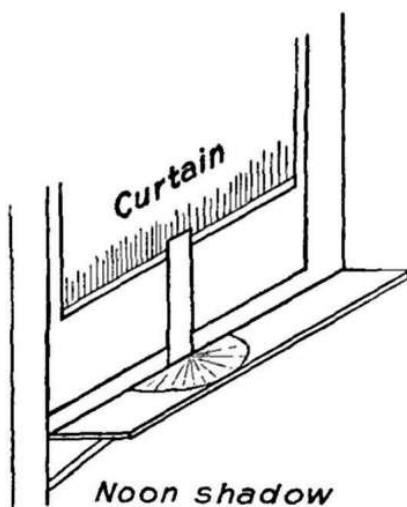


Fig. 3

1 o'clock, P.M. (see Fig. 3).

Carefully cut from cardboard a semicircle and mark the degrees, beginning with the middle radius as zero. Fasten this upon the window sill with the zero meridian coinciding with the north-south line. Note accurately the clock time when the shadow from the perpendicular board crosses the line, also where the shadow is at twelve o'clock. Record these facts with the date and preserve as Exhibit 3. Continue the observations every few days.

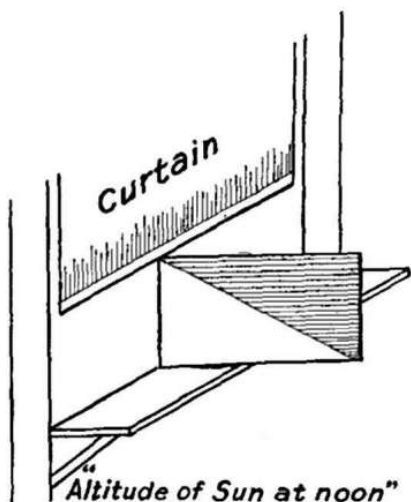


Fig. 4

The Sun's Meridian Altitude. When the shadow is due north, carefully measure the angle formed by the shadow and a level line. The simplest way is to draw the window shade down to the top of a sheet of cardboard placed very nearly north and south with the bottom level and then draw the shadow line, the lower acute angle being the one sought (see Fig. 4). Another way is to drive a pin in the side of the window casing, or in the edge of the vertical board (Fig. 3); fasten a

thread to it and connect the other end of the thread to a point on the sill where the shadow falls. A still better method is shown on p. 172.

Since the shadow is north, the sun is as high in the sky as it will get during the day, and the angle thus measured gives the highest altitude of the sun for the day. Record the measurement of the angle with the date as Exhibit 4. Continue these records

from week to week, especially noting the angle on one of the following dates: March 21, June 22, September 23, December 22. This angle on March 21 or September 23, if subtracted from 90° , will equal the latitude* of the observer.

A FEW TERMS EXPLAINED

Centrifugal Force.

The literal meaning of the word suggests its current meaning. It comes from the Latin *centrum*, center; and *fugere*, to flee. A centrifugal force is one directed away from a center. When a stone is whirled at the end of a string, the pull which the stone gives the string is called centrifugal force.

Because of the inertia of the stone, the whirling motion given to it by the arm tends to make it fly off in a straight line (Fig. 5),—and this it will do if the string breaks. The measure of the centrifugal force is the tension on the string. If the string be fastened at the end of a spring scale and the stone whirled, the scale will show the amount of the centrifugal force which is given the stone by the arm that whirls it. The amount of this force† (C) varies with the mass of the body (m), its velocity (v), and the radius of the

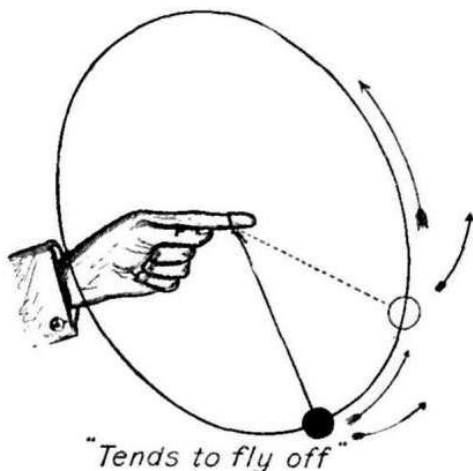


Fig. 5

*This is explained on p. 170.

†On the use of symbols, such as C for centrifugal force, ϕ for latitude, etc., see Appendix, p. 306.

circle (r) in which it moves, in the following ratio:

$$C = \frac{mv^2}{r}.$$

The instant that the speed becomes such that the available strength of the string is less than the value of $\frac{mv^2}{r}$, however slightly, the stone will cease to follow the curve and will immediately take a motion at a uniform speed in the straight line with which its motion happened to coincide at that instant (a tangent to the circle at the point reached at that moment).

Centrifugal Force on the Surface of the Earth. The rotating earth imparts to every portion of it, save along the axis, a centrifugal force which varies according to the foregoing formula, r being the distance to the axis, or the radius of the parallel. It is obvious that on the surface of the earth the centrifugal force due to its rotation is greatest at the equator and zero at the poles.

At the equator centrifugal force (C) amounts to about $\frac{1}{289}$ that of the earth's attraction (g), and thus an object there which weighs 288 pounds is lightened just one pound by centrifugal force, that is, it would weigh 289 pounds were the earth at rest. At latitude 30° , $C = \frac{g}{385}$ (that is, centrifugal force is $\frac{1}{385}$ the force of the earth's attraction); at 45° , $C = \frac{g}{578}$; at 60° , $C = \frac{g}{1156}$.

For any latitude the "lightening effect" of centrifugal force due to the earth's rotation equals $\frac{g}{289}$ times the square of the cosine of the latitude ($C = \frac{g}{289} \times \cos^2 \phi$). By referring to the table of cosines in the Appendix, the student can easily calculate the "lightening" influence of centrifugal force at his own latitude.

For example, say the latitude of the observer is 40° .

$$\text{Cosine } 40^\circ = .7660. \quad \frac{g}{289} \times .7660^2 = \frac{g}{492}.$$

Thus the earth's attraction for an object on its surface at latitude 40° is 492 times as great as centrifugal force there, and an object weighing 491 pounds at that latitude would weigh one pound more were the earth at rest.*

Centripetal Force. A centripetal (*centrum*, center; *petere*, to seek) force is one directed toward a center, that is, at right angles to the direction of motion of a body. To distinguish between centrifugal force and centripetal force, the student should realize that forces never occur singly but only in pairs and that in any force action there are always *two bodies* concerned. Name them *A* and *B*. Suppose *A* pushes or pulls *B* with a certain strength. This cannot occur except *B* pushes or pulls *A* by the same amount and in the opposite direction. This is only a simple way of stating Newton's third law that to every action (*A* on *B*) there corresponds an equal and opposite reaction (*B* on *A*).

Centrifugal force is the *reaction* of the body against the centripetal force which holds it in a curved path, and it must always exactly equal the centripetal force. In the case of a stone whirled at the end of a string, the necessary force which the string exerts on the stone to keep it in a curved path is centripetal force, and the reaction of the stone upon the string is centrifugal force.

The formulas for centripetal force are exactly the same as those for centrifugal force. Owing to the rotation of the earth, a body at the equator describes a circle with uniform speed. The attraction of the earth supplies the centripetal force required to hold it in the circle. The earth's attraction is greatly in excess

*These calculations are based upon a spherical earth and make no allowances for the oblateness.

of that which is required, being, in fact, 289 times the amount needed. *The centripetal force in this case is that portion of the attraction which is used to hold the object in the circular course.* The excess is what we call the weight of the body or the force of gravity.

If, therefore, a spring balance suspending a body at the equator shows 288 pounds, we infer that the earth really pulls it with a force of 289 pounds, but one pound of this pull is expended in changing the direction of the motion of the body, that is, the value of centripetal force is one pound. The body pulls the earth to the same extent, that is, the centrifugal force is also one pound. At the poles neither centripetal nor centrifugal force is exerted upon bodies and hence the weight of a body there is the full measure of the attraction of the earth.

Gravitation.

Gravitation is the all-pervasive force by virtue of which every particle of matter in the universe is constantly drawing toward itself every other particle of matter, however distant. The amount of this attractive force existing between two bodies depends upon

(1) the amount of matter in them, and (2) the distance they are apart.

There are thus two laws of gravitation. The first law, the greater the mass, or amount of matter, the greater the attrac-

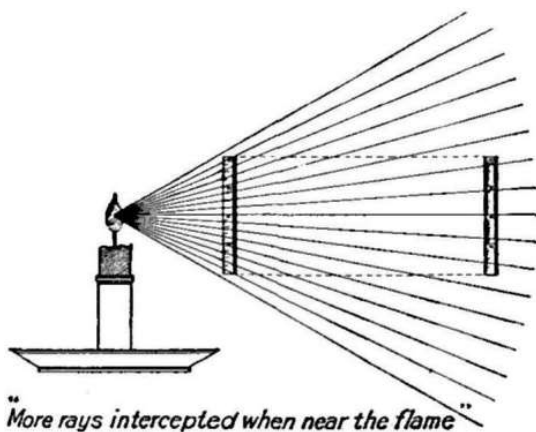


Fig. 6

tion, is due to the fact that each particle of matter has its own independent attractive force, and the more there are of the particles, the greater is the combined attraction.

The Second Law Explained. In general terms the law is that the nearer an object is, the greater is its attractive force. Just as the heat and light of a flame are greater the nearer one gets to it (Fig. 6), because more rays are intercepted, so the nearer an object is, the greater is its attraction. The ratio of the increase of the power of gravitation as distance decreases, may be seen from Figures 7 and 8.

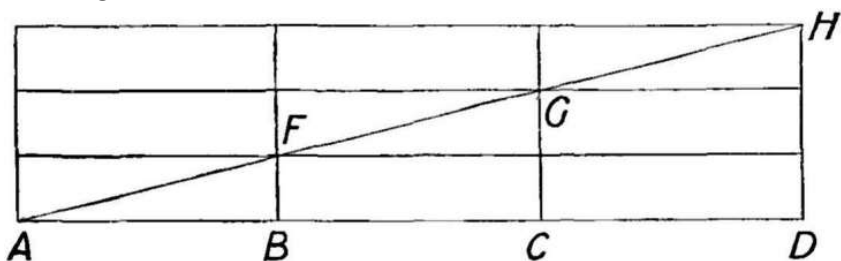


Fig. 7

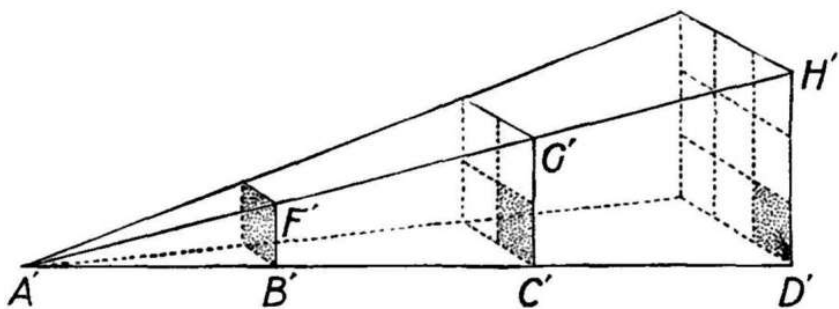


Fig. 8

Two lines, AD and AH (Fig. 7), are twice as far apart at C as at B because twice as far away; three times as far apart at D as at B because three times as far away, etc. Now light radiates out in every direction, so that light coming from point A' (Fig. 8),

when it reaches B' will be spread over the square of $B'F'$; at C' , on the square $C'G'$; at D' on the square $D'H'$, etc. C' being twice as far away from A' as B' , the side $C'G'$ is twice that of $B'F'$, as we observed in Fig. 7, and its square is four times as great. Line $D'H'$ is three times as far away, is three times as long, and its square is nine times as great. The light being spread over more space in the more distant objects, it will light up a given area less. The square at B' receives all the light within the four radii, the same square at C' receives one fourth of it, at D' one ninth, etc. The amount of light *decreases as the square of the distance increases*. The force of gravitation is exerted in every direction and varies in exactly the same way. Thus the second law of gravitation is that the force varies inversely as the square of the distance.

Gravity. The earth's attractive influence is called *gravity*. The weight of an object is simply the measure of the force of gravity. An object on or above the surface of the earth weighs less as it is moved away from the center of gravity.* It is difficult to realize that what we call the weight of an object is simply the excess of attraction which the earth possesses for it as compared with other forces acting upon it, and that it is a purely relative affair, the same object having a different weight in different places in the solar system. Thus the same pound-weight taken from the earth to the sun's surface would weigh 27 pounds there, only one sixth of a pound at the surface of the moon, over $2\frac{1}{2}$ pounds on Jupiter, etc. If the earth were more dense, objects would weigh more on the surface. Thus if the earth retained its present size but contained as much matter as the sun has, the strongest man in the world could not lift a silver half dollar, for it would then weigh over five tons. A pendulum clock would

*For a more accurate and detailed discussion of gravity, see p. 278.

then tick 575 times as fast. On the other hand, if the earth were no denser than the sun, a half dollar would weigh only a trifle more than a dime now weighs, and a pendulum clock would tick only half as fast.

From the table on p. 266 giving the masses and distances of the sun, moon, and principal planets, many interesting problems involving the laws of gravitation may be suggested. To illustrate, let us take the problem "What would you weigh if you were on the moon?"

Weight on the Moon. The mass of the moon, that is, the amount of matter in it, is $\frac{1}{81}$ that of the earth. Were it the same size as the earth and had this mass, one pound on the earth would weigh a little less than one eightieth of a pound there. According to the first law of gravitation we have this proportion:

$$1. \text{ Earth's attraction : Moon's attraction} :: 1 : \frac{1}{81}.$$

But the radius of the moon is 1081 miles, only a little more than one fourth that of the earth. Since a person on the moon would be so much nearer the center of gravity than he is on the earth, he would weigh much more there than here if the moon had the same mass as the earth. According to the second law of gravitation we have this proportion:

$$2. \text{ Earth's attraction : Moon's attraction} :: \frac{1}{4000^2} : \frac{1}{1081^2}.$$

We have then the two proportions:

$$1. \text{ Att. Earth : Att. Moon} :: 1 : \frac{1}{81}.$$

$$2. \text{ Att. Earth : Att. Moon} :: \frac{1}{4000^2} : \frac{1}{1081^2}.$$

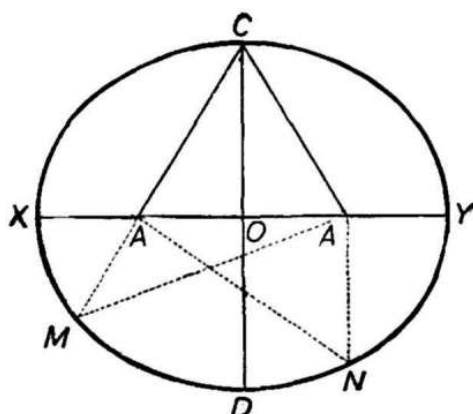
Combining these by multiplying, we get

$$\text{Att. Earth : Att. Moon} :: 6 : 1.$$

Therefore six pounds on the earth would weigh only one pound on the moon. Your weight, then, divided by six, represents what it would be on the moon. There you could jump six times as high—if you could live to jump at all on that cold and almost airless satellite (see p. 263).

The Sphere, Circle, and Ellipse. A *sphere* is a solid bounded by a curved surface all points of which are equally distant from a point within called the center.

A *circle* is a plane figure bounded by a curved line all points of which are equally distant from a point within called the center. In geography what we commonly call circles such as the equator, parallels, and meridians, are really only the circumferences of circles. Wherever used in this book, unless otherwise stated, the term circle refers to the circumference.



Ellipse.
A & A', Foci. C D, Minor Axis.
XY, Major Axis. A to A', Focal
Distance. $AM + A'M = AN + A'N$

Fig. 9

Every circle is conceived to be divided into 360 equal parts called degrees. The greater the size of the circle, the greater is the length of each degree. A *radius* of a circle or of a sphere is a straight line from the boundary to the center. Two radii forming a straight line constitute a *diameter*.

Circles on a sphere dividing it into two hemispheres are called *great circles*. Circles on a sphere dividing it into unequal parts

are called *small circles*.

All great circles on the same sphere bisect each other, regardless of the angle at which they cross one another. That this may be clearly seen, with a globe before you test these two propositions:

a. A point 180° in any direction from one point in a great circle must lie in the same circle.

b. Two great circles on the same sphere must cross somewhere, and the point 180° from the one where they cross, lies in both of the circles, thus each great circle divides the other into two equal parts.

An angle is the difference in direction of two lines which, if extended, would meet. Angles are measured by using the meeting point as the center of a circle and finding the fraction of the circle, or number of degrees of the circle, included between the lines. It is well to practice estimating different angles and then to test the accuracy of the estimates by reference to a graduated quadrant or circle having the degrees marked.

An *ellipse* is a closed plane curve such that the sum of the distances from one point in it to two fixed points within, called *foci*, is equal to the sum of the distances from any other point in it to the foci. The ellipse is a conic section formed by cutting a

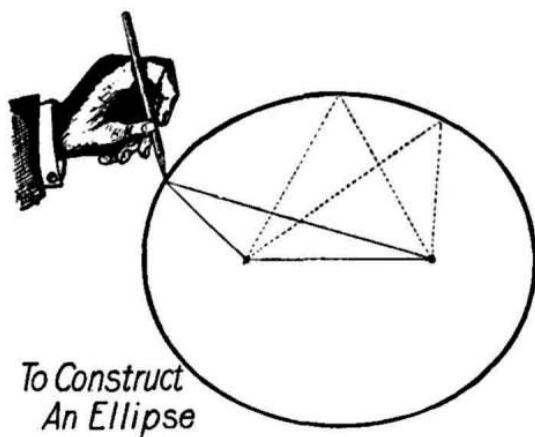


Fig. 10

passing obliquely through its opposite sides (see Ellipse in

Glossary).

To construct an ellipse, drive two pins at points for foci, say three inches apart. With a loop of non-elastic cord, say ten inches long, mark the boundary line as represented in Figure 10.

Orbit of the Earth. The orbit of the earth is an ellipse. To lay off an ellipse which shall quite correctly represent the shape of the earth's orbit, place pins one tenth of an inch apart and make a loop of string 12.2 inches long. This loop can easily be made by driving two pins 6.1 inches apart and tying a string looped around them.

Shape of the Earth. The earth is a spheroid, or a solid approaching a sphere (see Spheroid in Glossary). The diameter upon which it rotates is called the *axis*. The ends of the axis are its *poles*. Imaginary lines on the surface of the earth extending from pole to pole are called *meridians*.* While any number of meridians may be conceived of, we usually think of them as one degree apart. We say, for example, the ninetieth meridian, meaning the meridian ninety degrees from the prime or initial meridian. What kind of a circle is a meridian circle? Is it a true circle? Why?

The *equator* is a great circle midway between the poles.

Parallels are small circles parallel to the equator.

It is well for the student to bear in mind the fact that it is the earth's rotation on its axis that determines most of the foregoing facts. A sphere at rest would not have equator, meridians, etc.

*The term meridian is frequently used to designate a great circle passing through the poles. In this book such a circle is designated a *meridian circle*, since each meridian is numbered regardless of its opposite meridian.

CHAPTER II

THE FORM OF THE EARTH

THE EARTH A SPHERE

Circumnavigation. The statements commonly given as proofs of the spherical form of the earth would often apply as well to a cylinder or an egg-shaped or a disk-shaped body. "People have sailed around it," "The shadow of the earth as seen in the eclipse of the moon is always circular," etc., do not in themselves prove that the earth is a sphere. They might be true if the earth were a cylinder or had the shape of an egg. "But men have sailed around it in different directions." So might they a lemon-shaped body. To make a complete proof, we must show that men have sailed around it in practically every direction and have found no appreciable difference in the distances in the different directions.

Earth's Shadow always Circular. The shadow of the earth as seen in the lunar eclipse is always circular. But a dollar, a lemon, an egg, or a cylinder may be so placed as always to cast a circular shadow. When in addition to this statement it is shown that the earth presents many different sides toward the sun during different eclipses of the moon and the shadow is always circular, we have a proof positive, for nothing but a sphere casts a circular shadow when in many different positions. The fact that eclipses of the moon are seen in different seasons and at different times of day is abundant proof that practically all sides of the earth are turned toward the sun during different

eclipses.

Almost Uniform Surface Gravity. An object has almost exactly the same weight in different parts of the earth (that is, on the surface), showing a practically common distance from different points on the earth's surface to the center of gravity. This is ascertained, not by carrying an object all over the earth and weighing it with a pair of spring scales (why not balances?); but by noting the time of the swing of the pendulum, for the rate of its swing varies according to the force of gravity.

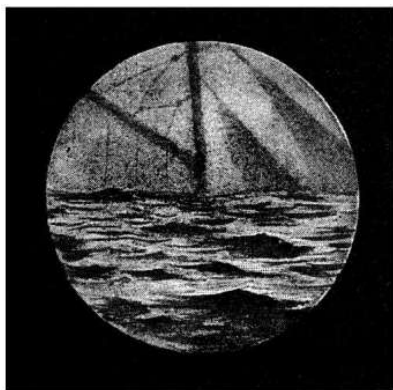


**Fig. 12. Water distinct.
Rigging ill-defined.**

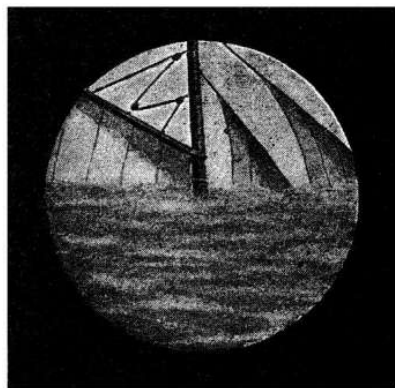


**Fig. 11. Ship's rigging
distinct. Water hazy.**

Telescopic Observations.

If we look through a telescope at a distant object over a level surface, such as a body of water, the lower part is hidden by the intervening curved surface. (Figs. 11, 12.) This has been observed in many different places, and the rate of curvature seems uniform everywhere and in every direction. Persons ascending in balloons or living on high elevations note the appreciably earlier time of sunrise or later time of

sunset at the higher elevation.

Shifting of Stars and Difference in Time. The proof which first demonstrated the curvature of the earth, and one which the student should clearly understand, is the disappearance of stars from the southern horizon and the rising higher of stars from the northern horizon to persons traveling north, and the sinking of northern stars and the rising of southern stars to south-bound travelers. After people had traveled far enough north and south to make an appreciable difference in the position of stars, they observed this apparent rising and sinking of the sky. Now two travelers, one going north and the other going south, will see the sky apparently elevated and depressed at the same time; that is, the portion of the sky that is rising for one will be sinking for the other. Since it is impossible that the stars be both rising and sinking at the same time, only one conclusion can follow,—the movement of the stars is apparent, and the path traveled north and south must be curved.

Owing to the rotation of the earth one sees the same stars in different positions in the sky east and west, so the proof just given simply shows that the earth is curved in a north and south direction. Only when timepieces were invented which could carry the time of one place to different portions of the earth could the apparent positions of the stars prove the curvature of the earth east and west. By means of the telegraph and telephone we have most excellent proof that the earth is curved east and west.

If the earth were flat, when it is sunrise at Philadelphia it would be sunrise also at St. Louis and Denver. Sun rays extending to these places which are so near together as compared with the tremendous distance of the sun, over ninety millions of miles away, would be almost parallel on the earth and would strike these points at about the same angle. But we know from the many daily messages between these cities that sun time in

Philadelphia is an hour later than it is in St. Louis and two hours later than in Denver.

When we know that the curvature of the earth north and south as observed by the general and practically uniform rising and sinking of the stars to north-bound and south-bound travelers is the same as the curvature east and west as shown by the difference in time of places east and west, we have an excellent proof that the earth is a sphere.

Actual Measurement. Actual measurement in many different places and in nearly every direction shows a practically uniform curvature in the different directions. In digging canals and laying watermains, an allowance must always be made for the curvature of the earth; also in surveying, as we shall notice more explicitly farther on.

A simple rule for finding the amount of curvature for any given distance is the following:

Square the number of miles representing the distance, and two thirds of this number represents in feet the departure from a straight line.

Suppose the distance is 1 mile. That number squared is 1, and two thirds of that number of feet is 8 inches. Thus an allowance of 8 inches must be made for 1 mile. If the distance is 2 miles, that number squared is 4, and two thirds of 4 feet is 2 feet, 8 inches. An object, then, 1 mile away sinks 8 inches below the level line, and at 2 miles it is below 2 feet, 8 inches.

To find the distance, the height from a level line being given, we have the converse of the foregoing rule:

Multiply the number representing the height in feet by $1\frac{1}{2}$, and the square root of this product represents the number of miles distant the object is situated.

The following table is based upon the more accurate formula:

$$\text{Distance (miles)} = 1.317\sqrt{\text{height (feet)}}.$$

Ht. ft.	Dist. miles	Ht. ft.	Dist. miles	Ht. ft.	Dist. miles
1	1.32	50	9.31	170	17.17
2	1.86	55	9.77	180	17.67
3	2.28	60	10.20	190	18.15
4	2.63	65	10.62	200	18.63
5	2.94	70	11.02	300	22.81
6	3.23	75	11.40	400	26.34
7	3.48	80	11.78	500	29.45
8	3.73	85	12.14	600	32.26
9	3.95	90	12.49	700	34.84
10	4.16	95	12.84	800	37.25
15	5.10	100	13.17	900	39.51
20	5.89	110	13.81	1000	41.65
25	6.59	120	14.43	2000	58.90
30	7.21	130	15.02	3000	72.13
35	7.79	140	15.58	4000	83.30
40	8.33	150	16.13	5000	93.10
45	8.83	160	16.66	Mile	95.70

THE EARTH AN OBLATE SPHEROID

Richer's Discovery. In the year 1672 John Richer, the astronomer to the Royal Academy of Sciences of Paris, was sent by Louis XIV to the island of Cayenne to make certain astronomical observations. His Parisian clock had its pendulum, slightly over 39 inches long, regulated to beat seconds. Shortly after his arrival at Cayenne, he noticed that the clock was losing time, about two and a half minutes a day. Gravity, evidently, did not act with so much force near the equator as it did at Paris. The astronomer found it necessary to shorten the pendulum nearly a quarter of an inch to get it to swing fast enough.

Richer reported these interesting facts to his colleagues at Paris, and it aroused much discussion. At first it was thought that greater centrifugal force at the equator, counteracting the

earth's attraction more there than elsewhere, was the explanation. The difference in the force of gravity, however, was soon discovered to be too great to be thus accounted for. The only other conclusion was that Cayenne must be farther from the center of gravity than Paris (see the discussion of Gravity, Appendix, p. 278; also Historical Sketch, pp. 272–274).

Repeated experiments show it to be a general fact that pendulums swing faster on the surface of the earth as one approaches the poles. Careful measurements of arcs of meridians prove beyond question that the earth is flattened toward the poles, somewhat like an oblate spheroid. Further evidence is found in the fact that the sun and planets, so far as ascertained, show this same flattening.

Cause of Oblateness. The cause of the oblateness is the rotation of the body, its flattening effects being more marked in earlier plastic stages, as the earth and other planets are generally believed to have been at one time. The reason why rotation causes an equatorial bulging is not difficult to understand. Centrifugal force increases away from the poles toward the equator and gives a lifting or lightening influence to portions on the surface. If the earth were a sphere, an object weighting 289 pounds at the poles would be lightened just one pound if carried to the swiftly rotating equator (see p. 279). The form given the earth by its rotation is called an oblate spheroid or an ellipsoid of rotation.

Amount of Oblateness. To represent a meridian circle accurately, we should represent the polar diameter about $\frac{1}{300}$ part shorter than the equatorial diameter. That this difference is not perceptible to the unaided eye will be apparent if the construction of such a figure is attempted, say ten inches in diameter in one direction and $\frac{1}{30}$ of an inch less in the opposite direction. The oblateness of Saturn is easily perceptible,

being thirty times as great as that of the earth, or one tenth (see p. 258). Thus an ellipsoid nine inches in polar diameter (minor axis) and ten inches in equatorial diameter (major axis) would represent the form of that planet.

Although the oblateness of the earth seems slight when represented on a small scale and for most purposes may be ignored, it is nevertheless of vast importance in many problems in surveying, astronomy, and other subjects. Under the discussion of latitude it will be shown how this oblateness makes a difference in the lengths of degrees of latitude, and in the Appendix it is shown how this equatorial bulging shortens the length of the year and changes the inclination of the earth's axis (see Precession of the Equinoxes and Motions of the Earth's Axis).

Dimensions of the Spheroid. It is of very great importance in many ways that astronomers and surveyors know as exactly as possible the dimensions of the spheroid. Many men have made estimates based upon astronomical facts, pendulum experiments and careful surveys, as to the equatorial and polar diameters of the earth. Perhaps the most widely used is the one made by A. R. Clarke, for many years at the head of the English Ordnance Survey, known as the Clarke Spheroid of 1866.

CLARKE SPHEROID OF 1866.

A.	Equatorial diameter	7,926.614 miles
B.	Polar diameter	7,899.742 miles
	Oblateness $\frac{A - B}{A}$	$\frac{1}{295}$

It is upon this spheroid of reference that all of the work of the United States Geological Survey and of the United States Coast and Geodetic Survey is based, and upon which most of the dimensions given in this book are determined.

In 1878 Mr. Clarke made a recalculation, based upon additional information, and gave the following dimensions, though it

is doubtful whether these approximations are any more nearly correct than those of 1866.

CLARKE SPHEROID OF 1878.

A. Equatorial diameter	7,926.592 miles
B. Polar diameter	7,899.580 miles
Oblateness $\frac{A - B}{A}$	$\frac{1}{293.46}$

Another standard spheroid of reference often referred to, and one used by the United States Governmental Surveys before 1880, when the Clarke spheroid was adopted, was calculated by the distinguished Prussian astronomer, F. H. Bessel, and is called the

BESSEL SPHEROID OF 1841.

A. Equatorial diameter	7,925.446 miles
B. Polar diameter	7,898.954 miles
Oblateness $\frac{A - B}{A}$	$\frac{1}{299.16}$

Many careful pendulum tests and a great amount of scientific triangulation surveys of long arcs of parallels and meridians within recent years have made available considerable data from which to determine the true dimensions of the spheroid. In 1900, the United States Coast and Geodetic Survey completed the measurement of an arc across the United States along the 39th parallel from Cape May, New Jersey, to Point Arena, California, through $48^{\circ} 46'$ of longitude, or a distance of about 2,625 miles. This is the most extensive piece of geodetic surveying ever undertaken by any nation and was so carefully done that the total amount of probable error does not amount to more than about eighty-five feet. A long arc has been surveyed diagonally from Calais, Maine, to New Orleans, Louisiana, through $15^{\circ} 1'$ of latitude and $22^{\circ} 47'$ of longitude, a distance of 1,623 miles.

Another long arc will soon be completed along the 98th meridian across the United States. Many shorter arcs have also been surveyed in this country.

The English government undertook in 1899 the gigantic task of measuring the arc of a meridian extending the entire length of Africa, from Cape Town to Alexandria. This will be, when completed, 65° long, about half on each side of the equator, and will be of great value in determining the oblateness. Russia and Sweden have lately completed the measurement of an arc of $4^{\circ} 30'$ on the island of Spitzbergen, which from its high latitude, 76° to $80^{\circ} 30' N.$, makes it peculiarly valuable. Large arcs have been measured in India, Russia, France, and other countries, so that there are now available many times as much data from which the form and dimensions of the earth may be determined as Clarke or Bessel had.

The late Mr. Charles A. Schott, of the United States Coast and Geodetic Survey, in discussing the survey of the 39th parallel, with which he was closely identified, said:*

“Abundant additional means for improving the existing deductions concerning the earth’s figure are now at hand, and it is perhaps not too much to expect that the International Geodetic Association may find it desirable in the near future to attempt a new combination of all available arc measures, especially since the two large arcs of the parallel, that between Ireland and Poland and that of the United States of America, cannot fail to have a paramount influence in a new general discussion.”

A spheroid is a solid nearly spherical. An oblate spheroid is one flattened toward the poles of its axis of rotation. The earth is commonly spoken of as a sphere. It would be more nearly

*In his Transcontinental Triangulation and the American Arc of the Parallel.

correct to say it is an oblate spheroid. This, however, is not strictly accurate, as is shown in the succeeding discussion.

THE EARTH A GEOID

Conditions Producing Irregularities. If the earth had been made up of the same kinds of material uniformly distributed throughout its mass, it would probably have assumed, because of its rotation, the form of a regular oblate spheroid. But the earth has various materials unevenly distributed in it, and this has led to many slight variations from regularity in form.

Equator Elliptical. Pendulum experiments and measurements indicate not only that meridians are elliptical but that the equator itself may be slightly elliptical, its longest axis passing through the earth from 15° E. to 165° W. and its shortest axis from 105° E. to 75° W. The amount of this oblateness of the equator is estimated at about $\frac{1}{4,000}$ or a difference of two miles in the lengths of these two diameters of the equator. Thus the meridian circle passing through central Africa and central Europe (15° E.) and around near Behring Strait (165° W.) may be slightly more oblate than the other meridian circles, the one which is most nearly circular passing through central Asia (105° E.), eastern North America, and western South America (75° W.).

United States Curved Unequally. It is interesting to note that the dimensions of the degrees of the long arc of the 39th parallel surveyed in the United States bear out to a remarkable extent the theory that the earth is slightly flattened longitudinally, making it even more than that just given, which was calculated by Sir John Herschel and A. R. Clarke. The average length of degrees of longitude from the Atlantic coast for

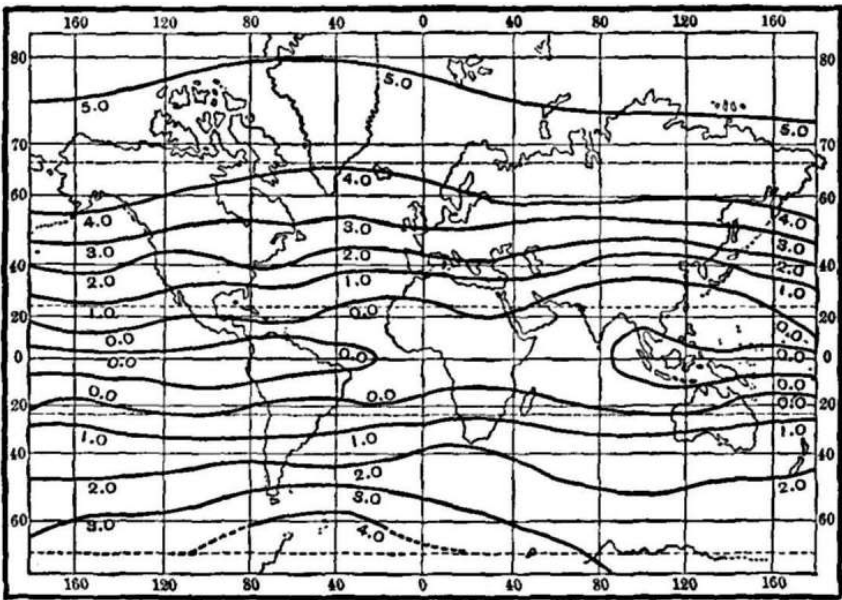


Fig. 13. Gravimetric lines showing variation in force of gravity

the first 1,500 miles corresponds closely to the Clarke table, and thus those degrees are longer, and the rest of the arc corresponds closely to the Bessel table and shows shorter degrees.

	Diff. in long.	Length of 1°	Clarke	Bessel
Cape May to Wallace (Kansas)	26.661°	53.829 mi.	53.828 mi.	
Wallace to Uriah (Calif.)	21.618°	53.822 mi.		53.821 mi.

Earth not an Ellipsoid of Three Unequal Axes. This oblateness of the meridians and oblateness of the equator led some to treat the earth as an ellipsoid of three unequal axes: (1) the longest equatorial axis, (2) the shortest equatorial axis,

and (3) the polar axis. It has been shown, however, that meridians are not true ellipses, for the amount of flattening northward is not quite the same as the amount southward, and the mathematical center of the earth is not exactly in the plane of the equator.

Geoid Defined. The term *geoid*, which means “like the earth,” is now applied to *that figure which most nearly corresponds to the true shape of the earth*. Mountains, valleys, and other slight deviations from evenness of surfaces are treated as departures from the geoid of reference. The following definition by Robert S. Woodward, President of the Carnegie Institution of Washington, very clearly explains what is meant by the geoid.*

“Imagine the mean sea level, or the surface of the sea freed from the undulations due to winds and to tides. This mean sea surface, which may be conceived to extend through the continents, is called the geoid. It does not coincide exactly with the earth’s spheroid, but is a slightly wavy surface lying partly above and partly below the spheroidal surface, by small but as yet not definitely known amounts. The determination of the geoid is now one of the most important problems of geophysics.”

An investigation is now in progress in the United States for determining a new geoid of reference upon a plan never followed hitherto. The following is a lucid description[†] of the plan by John F. Hayford, Inspector of Geodetic Work, United States Coast and Geodetic Survey.

Area Method of Determining Form of the Earth.

“The arc method of deducing the figure of the earth may be illustrated by supposing that a skilled workman to whom is given several stiff wires, each representing a geodetic arc, either of a parallel or a meridian, each bent to the radius deduced from

*Encyclopaedia Americana.

[†]Given at the International Geographic Congress, 1904.