A FIRST BOOK IN ALGEBRA

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PREFACE

In preparing this book, the author had especially in mind classes in the upper grades of grammar schools, though the work will be found equally well adapted to the needs of any classes of beginners.

The ideas which have guided in the treatment of the subject are the following: The study of algebra is a continuation of what the pupil has been doing for years, but it is expected that this new work will result in a knowledge of *general truths* about numbers, and an increased power of clear thinking. All the differences between this work and that pursued in arithmetic may be traced to the introduction of two new elements, namely, negative numbers and the representation of numbers by letters. The solution of problems is one of the most valuable portions of the work, in that it serves to develop the thought-power of the pupil at the same time that it broadens his knowledge of numbers and their relations. Powers are developed and habits formed only by persistent, long-continued practice.

Accordingly, in this book, it is taken for granted that the pupil knows what he may be reasonably expected to have learned from his study of arithmetic; abundant practice is given in the representation of numbers by letters, and great care is taken to make clear the meaning of the minus sign as applied to a single number, together with the modes of operating upon negative numbers; problems are given in every exercise in the book; and, instead of making a statement of what the child is to see in the illustrative example, questions are asked which shall lead him to find for himself that which he is to learn from the example.

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A FIRST BOOK IN ALGEBRA.

ALGEBRAIC NOTATION.

1. Algebra is so much like arithmetic that all that you know about addition, subtraction, multiplication, and division, the signs that you have been using and the ways of working out problems, will be very useful to you in this study. There are two things the introduction of which really makes all the difference between arithmetic and algebra. One of these is the use of *letters to represent numbers*, and you will see in the following exercises that this change makes the solution of problems much easier.

Exercise I.

Illustrative Example. The sum of two numbers is 60, and the greater is four times the less. What are the numbers?

Solution.

Let	x= the less number;
then	$4x$= the greater number,
and	$4x + x$=60,
or	$5x$=60;
therefore	x=12,
and	$4x$=48. The numbers are 12 and 48.

1. The greater of two numbers is twice the less, and the sum of the numbers is 129. What are the numbers?

2. A man bought a horse and carriage for $500, paying three times as much for the carriage as for the horse. How much did each cost?

3. Two brothers, counting their money, found that together they had $186, and that John had five times as much as Charles. How much had each?

4. Divide the number 64 into two parts so that one part shall be seven times the other.

5. A man walked 24 miles in a day. If he walked twice as far in the forenoon as in the afternoon, how far did he walk in the afternoon?

6. For 72 cents Martha bought some needles and thread, paying eight times as much for the thread as for the needles. How much did she pay for each?

7. In a school there are 672 pupils. If there are twice as many boys as girls, how many boys are there?

Illustrative Example. If the difference between two numbers is 48, and one number is five times the other, what are the numbers?

Solution.

Let	x= the less number;
then	$5x$= the greater number,
and	$5x - x$=48,
or	$4x$=48;
therefore	x=12,
and	$5x$=60.

The numbers are 12 and 60.

8. Find two numbers such that their difference is 250 and one is eleven times the other.

9. James gathered 12 quarts of nuts more than Henry gathered. How many did each gather if James gathered three times as many as Henry?

10. A house cost $2880 more than a lot of land, and five times the cost of the lot equals the cost of the house. What was the cost of each?

11. Mr. A. is 48 years older than his son, but he is only three times as old. How old is each?

12. Two farms differ by 250 acres, and one is six times as large as the other. How many acres in each?

13. William paid eight times as much for a dictionary as for a rhetoric. If the difference in price was $6.30, how much did he pay for each?

14. The sum of two numbers is 4256, and one is 37 times as great as the other. What are the numbers?

15. Aleck has 48 cents more than Arthur, and seven times Arthur's money equals Aleck's. How much has each?

16. The sum of the ages of a mother and daughter is 32 years, and the age of the mother is seven times that of the daughter. What is the age of each?

17. John's age is three times that of Mary, and he is 10 years older. What is the age of each?

Exercise 2.

Illustrative Example. There are three numbers whose sum is 96; the second is three times the first, and the third is four times the first. What are the numbers?

Solution.

$$\begin{aligned} \text{Let} \quad & x\text{=first number,} \\ & 3x\text{=second number,} \\ & 4x\text{=third number.} \\ x + 3x + & 4x = 96 \\ & 8x = 90 \\ & x = 12 \\ & 3x = 36 \\ & 4x = 48 \end{aligned}$$

The numbers are 12, 36, and 48.

1. A man bought a hat, a pair of boots, and a necktie for \$7.50; the hat cost four times as much as the necktie, and the boots cost five times as much as the necktie. What was the cost of each?

2. A man traveled 90 miles in three days. If he traveled twice as far the first day as he did the third, and three times as far the second day as the third, how far did he go each day?

3. James had 30 marbles. He gave a certain number to his sister, twice as many to his brother, and had three times as many left as he gave his sister. How many did each then have?

4. A farmer bought a horse, cow, and pig for \$90. If he paid three times as much for the cow as for the pig, and five times as much for the horse as for the pig, what was the price of each?

5. A had seven times as many apples, and B three times as many as C had. If they all together had 55 apples, how many had each?

6. The difference between two numbers is 36, and one is four times the other. What are the numbers?

7. In a company of 48 people there is one man to each five women. How many are there of each?

8. A man left \$1400 to be distributed among three sons in such a way that James was to receive double what John received, and John double what Henry received. How much did each receive?

9. A field containing 45,000 feet was divided into three lots so that the second lot was three times the first, and the third twice the second. How large was each lot?

10. There are 120 pigeons in three flocks. In the second there are three times as many as in the first, and in the third as many as in the first and second combined. How many pigeons in each flock?

11. Divide 209 into three parts so that the first part shall be five times the second, and the second three times the third.

12. Three men, A, B, and C, earned \$110; A earned four times as much as B, and C as much as both A and B. How much did each earn?

13. A farmer bought a horse, a cow, and a calf for \$72; the cow cost twice as much as the calf, and the horse three times as much as the cow. What was the cost of each?

14. A cistern, containing 1200 gallons of water, is emptied by two pipes in two hours. One pipe discharges three times as many gallons per hour as the other. How many gallons does each pipe discharge in an hour?

15. A butcher bought a cow and a lamb, paying six times as much for the cow as for the lamb, and the difference of the prices was \$25. How much did he pay for each?

16. A grocer sold one pound of tea and two pounds of coffee for \$1.50, and the price of the tea per pound was three times that of the coffee. What was the price of each?

17. By will Mrs. Cabot was to receive five times as much as her son Henry. If Henry received \$20,000 less than his mother, how much did each receive?

Exercise 3.

Illustrative Example. Divide the number 126 into two parts such that one part is 8 more than the other.

Solution

Let x=less part,

$x+8$=greater part.

$$x+x+8=126$$
$$2x+8=126$$
$$2x=118^{1}$$
$$x=59$$
$$x+8=67$$

The parts are 59 and 67.

1. In a class of 35 pupils there are 7 more girls than boys. How many are there of each?

[1] Where in arithmetic did you learn the principle applied in transposing the 8?

2. The sum of the ages of two brothers is 43 years, and one of them is 15 years older than the other. Find their ages.

3. At an election in which 1079 votes were cast the successful candidate had a majority of 95. How many votes did each of the two candidates receive?

4. Divide the number 70 into two parts, such that one part shall be 26 less than the other part.

5. John and Henry together have 143 marbles. If I should give Henry 15 more, he would have just as many as John. How many has each?

6. In a storehouse containing 57 barrels there are 3 less barrels of flour than of meal. How many of each?

7. A man whose herd of cows numbered 63 had 17 more Jerseys than Holsteins. How many had he of each?

8. Two men whose wages differ by 8 dollars receive both together $44 per month. How much does each receive?

9. Find two numbers whose sum is 99 and whose difference is 19.

10. The sum of three numbers is 56; the second is 3 more than the first, and the third 5 more than the first. What are the numbers?

11. Divide 62 into three parts such that the first part is 4 more than the second, and the third 7 more than the second.

12. Three men together received $34,200; if the second received $1500 more than the first, and the third $1200 more than the second, how much did each receive?

13. Divide 65 into three parts such that the second part is 17 more than the first part, and the third 15 less than the first.

14. A man had 95 sheep in three flocks. In the first flock there were 23 more than in the second, and in the third flock 12 less than in the second. How many sheep in each flock?

15. In an election, in which 1073 ballots were cast, Mr. A receives 97 votes less than Mr. B, and Mr. C 120 votes more than Mr. B. How many votes did each receive?

16. A man owns three farms. In the first there are 5 acres more than in the second and 7 acres less than in the third. If there are 53 acres in all the farms together, how many acres are there in each farm?

17. Divide 111 into three parts so that the first part shall be 16 more than the second and 19 less than the third.

18. Three firms lost $118,000 by fire. The second firm lost $6000 less than the first and $20,000 more than the third. What was each firm's loss?

Exercise 4.

Illustrative Example. The sum of two numbers is 25, and the larger is 3 less than three times the smaller. What are the numbers?

Solution.

$$\begin{aligned} \text{Let} \qquad x &= \text{smaller number,} \\ 3x - 3 &= \text{larger number.} \\ x + 3x - 3 &= 25 \\ 4x - 3 &= 25 \\ 4x &= 28^{2} \\ x &= 7 \\ 3x - 3 &= 18 \end{aligned}$$

The numbers are 7 and 18.

1. Charles and Henry together have 49 marbles, and Charles has twice as many as Henry and 4 more. How many marbles has each?

2. In an orchard containing 33 trees the number of pear trees is 5 more than three times the number of apple trees. How many are there of each kind?

3. John and Mary gathered 23 quarts of nuts. John gathered 2 quarts more than twice as many as Mary. How many quarts did each gather?

4. To the double of a number I add 17 and obtain as a result 147. What is the number?

5. To four times a number I add 23 and obtain 95. What is the number?

6. From three times a number I take 25 and obtain 47. What is the number?

7. Find a number which being multiplied by 5 and having 14 added to the product will equal 69.

8. I bought some tea and coffee for $10.39. If I paid for the tea 61 cents more than five times as much as for the coffee, how much did I pay for each?

9. Two houses together contain 48 rooms. If the second house has 3 more than twice as many rooms as the first, how many rooms has each house?

Illustrative Example. Mr. Y gave $6 to his three boys. To the second he gave 25 cents more than to the third, and to the first three times as much as to the second. How much did each receive?

Solution.

[2] Is the same principle applied here that is applied on page 12?

Let x=number of cents third boy received,

$x+25$=number of cents second boy received,

$3x+75$=number of cents first boy received.

$$x+x+25+3x+75=600$$
$$5x+100=600$$
$$5x=500$$
$$x=100$$
$$x+25=125$$
$$3x+75=375$$

1st boy received \$3.75,

2d boy received \$1.25,

3d boy received \$1.00.

10. Divide the number 23 into three parts, such that the second is 1 more than the first, and the third is twice the second.

11. Divide the number 137 into three parts, such that the second shall be 3 more than the first, and the third five times the second.

12. Mr. Ames builds three houses. The first cost \$2000 more than the second, and the third twice as much as the first. If they all together cost \$18,000, what was the cost of each house?

13. An artist, who had painted three pictures, charged \$18 more for the second than the first, and three times as much for the third as the second. If he received \$322 for the three, what was the price of each picture?

14. Three men, A, B, and C, invest \$47,000 in business. B puts in \$500 more than twice as much as A, and C puts in three times as much as B. How many dollars does each put into the business?

15. In three lots of land there are 80,750 feet. The second lot contains 250 feet more than three times as much as the first lot, and the third lot contains twice as much as the second. What is the size of each lot?

16. A man leaves by his will \$225,000 to be divided as follows: his son to receive \$10,000 less than twice as much as the daughter, and the widow four times as much as the son. What was the share of each?

17. A man and his two sons picked 25 quarts of berries. The older son picked 5 quarts less than three times as many as the younger son, and the father picked twice as many as the older son. How many quarts did each pick?

18. Three brothers have 574 stamps. John has 15 less than Henry, and Thomas has 4 more than John. How many has each?

Exercise 5

.

Illustrative Example. Arthur bought some apples and twice as many oranges for 78 cents. The apples cost 3 cents apiece, and the oranges 5 cents apiece. How many of each did he buy?

Solution.

$$\begin{array}{lrcl} \text{Let} & x & = & \text{number of apples,} \\ & 2x & = & \text{number of oranges,} \\ & 3x & = & \text{cost of apples,} \\ & 10x & = & \text{cost of oranges.} \\ & 3x + 10x & = & 78 \\ & 13x & = & 78 \\ & x & = & 6 \\ & 2x & = & 12 \end{array}$$

Arthur bought 6 apples and 12 oranges.

1. Mary bought some blue ribbon at 7 cents a yard, and three times as much white ribbon at 5 cents a yard, paying $1.10 for the whole. How many yards of each kind did she buy?

2. Twice a certain number added to five times the double of that number gives for the sum 36. What is the number?

3. Mr. James Cobb walked a certain length of time at the rate of 4 miles an hour, and then rode four times as long at the rate of 10 miles an hour, to finish a journey of 88 miles. How long did he walk and how long did he ride?

4. A man bought 3 books and 2 lamps for $14. The price of a lamp was twice that of a book. What was the cost of each?

5. George bought an equal number of apples, oranges, and bananas for $1.08; each apple cost 2 cents, each orange 4 cents, and each banana 3 cents. How many of each did he buy?

6. I bought some 2-cent stamps and twice as many 5-cent stamps, paying for the whole $1.44. How many stamps of each kind did I buy?

7. I bought 2 pounds of coffee and 1 pound of tea for $1.31; the price of a pound of tea was equal to that of 2 pounds of coffee and 3 cents more. What was the cost of each per pound?

8. A lady bought 2 pounds of crackers and 3 pounds of gingersnaps for $1.11. If a pound of gingersnaps cost 7 cents more than a pound of crackers, what was the price of each?

9. A man bought 3 lamps and 2 vases for \$6. If a vase cost 50 cents less than 2 lamps, what was the price of each?

10. I sold three houses, of equal value, and a barn for \$16,800. If the barn brought \$1200 less than a house, what was the price of each?

11. Five lots, two of one size and three of another, aggregate 63,000 feet. Each of the two is 1500 feet larger than each of the three. What is the size of the lots?

12. Four pumps, two of one size and two of another, can pump 106 gallons per minute. If the smaller pumps 5 gallons less per minute than the larger, how much does each pump per minute?

13. Johnson and May enter into a partnership in which Johnson's interest is four times as great as May's. Johnson's profit was \$4500 more than May's profit. What was the profit of each?

14. Three electric cars are carrying 79 persons. In the first car there are 17 more people than in the second and 15 less than in the third. How many persons in each car?

15. Divide 71 into three parts so that the second part shall be 5 more than four times the first part, and the third part three times the second.

16. I bought a certain number of barrels of apples and three times as many boxes of oranges for \$33. I paid \$2 a barrel for the apples, and \$3 a box for the oranges. How many of each did I buy?

17. Divide the number 288 into three parts, so that the third part shall be twice the second, and the second five times the first.

18. Find two numbers whose sum is 216 and whose difference is 48.

Exercise 6

.

Illustrative Example. What number added to twice itself and 40 more will make a sum equal to eight times the number?

Solution.

$$\begin{array}{rrcl} \text{Let} & x & = & \text{the number.} \\ & x+2x+40 & = & 8x \\ & 3x+40 & = & 8x \\ & 40 & = & 5x \\ & 8 & = & x \end{array}$$

The number is 8.

1. What number, being increased by 36, will be equal to ten times itself?

2. Find the number whose double increased by 28 will equal six times the number itself.

3. If John's age be multiplied by 5, and if 24 be added to the product, the sum will be seven times his age. What is his age?

4. A father gave his son four times as many dollars as he then had, and his mother gave him $25, when he found that he had nine times as many dollars as at first. How many dollars had he at first?

5. A man had a certain amount of money; he earned three times as much the next week and found $32. If he then had eight times as much as at first, how much had he at first?

6. A man, being asked how many sheep he had, said, "If you will give me 24 more than six times what I have now, I shall have ten times my present number." How many had he?

7. Divide the number 726 into two parts such that one shall be five times the other.

8. Find two numbers differing by 852, one of which is seven times the other.

9. A storekeeper received a certain amount the first month; the second month he received $50 less than three times as much, and the third month twice as much as the second month. In the three months he received $4850. What did he receive each month?

10. James is 3 years older than William, and twice James's age is equal to three times William's age. What is the age of each?

11. One boy has 10 more marbles than another boy. Three times the first boy's marbles equals five times the second boy's marbles. How many has each?

12. If I add 12 to a certain number, four times this second number will equal seven times the original number. What is the original number?

13. Four dozen oranges cost as much as 7 dozen apples, and a dozen oranges cost 15 cents more than a dozen apples. What is the price of each?

14. Two numbers differ by 6, and three times one number equals five times the other number. What are the numbers?

15. A man is 2 years older than his wife, and 15 times his age equals 16 times her age. What is the age of each?

16. A farmer pays just as much for 4 horses as he does for 6 cows. If a cow costs 15 dollars less than a horse, what is the cost of each?

17. What number is that which is 15 less than four times the number itself?

18. A man bought 12 pairs of boots and 6 suits of clothes for $168. If a suit of clothes cost $2 less than four times as much as a pair of boots, what was the price of each?

Exercise 7

.

Illustrative Example. Divide the number 72 into two parts such that one part shall be one-eighth of the other.

Solution.

$$\begin{array}{lrcl} \text{Let} & x & = & \text{greater part,} \\ & \frac{1}{8}x & = & \text{lesser part.} \\ & x+\frac{1}{8}x & = & 72 \\ & \frac{9}{8}x & = & 72 \\ & \frac{1}{8}x & = & 8 \\ & x & = & 64 \end{array}$$

The parts are 64 and 8.

1. Roger is one-fourth as old as his father, and the sum of their ages is 70 years. How old is each?

2. In a mixture of 360 bushels of grain, there is one-fifth as much corn as wheat. How many bushels of each?

3. A man bought a farm and buildings for $12,000. The buildings were valued at one-third as much as the farm. What was the value of each?

4. A bicyclist rode 105 miles in a day. If he rode one-half as far in the afternoon as in the forenoon, how far did he ride in each part of the day?

5. Two numbers differ by 675, and one is one-sixteenth of the other. What are the numbers?

6. What number is that which being diminished by one-seventh of itself will equal 162?

7. Jane is one-fifth as old as Mary, and the difference of their ages is 12 years. How old is each?

Illustrative Example. The half and fourth of a certain number are together equal to 75. What is the number?

Solution.

$$\begin{array}{lrcl} \text{Let} & x & = & \text{the number.} \\ & \frac{1}{2}x+\frac{1}{4}x & = & 75. \\ & \frac{3}{4}x & = & 75 \\ & \frac{1}{4}x & = & 25 \\ & x & = & 100 \end{array}$$