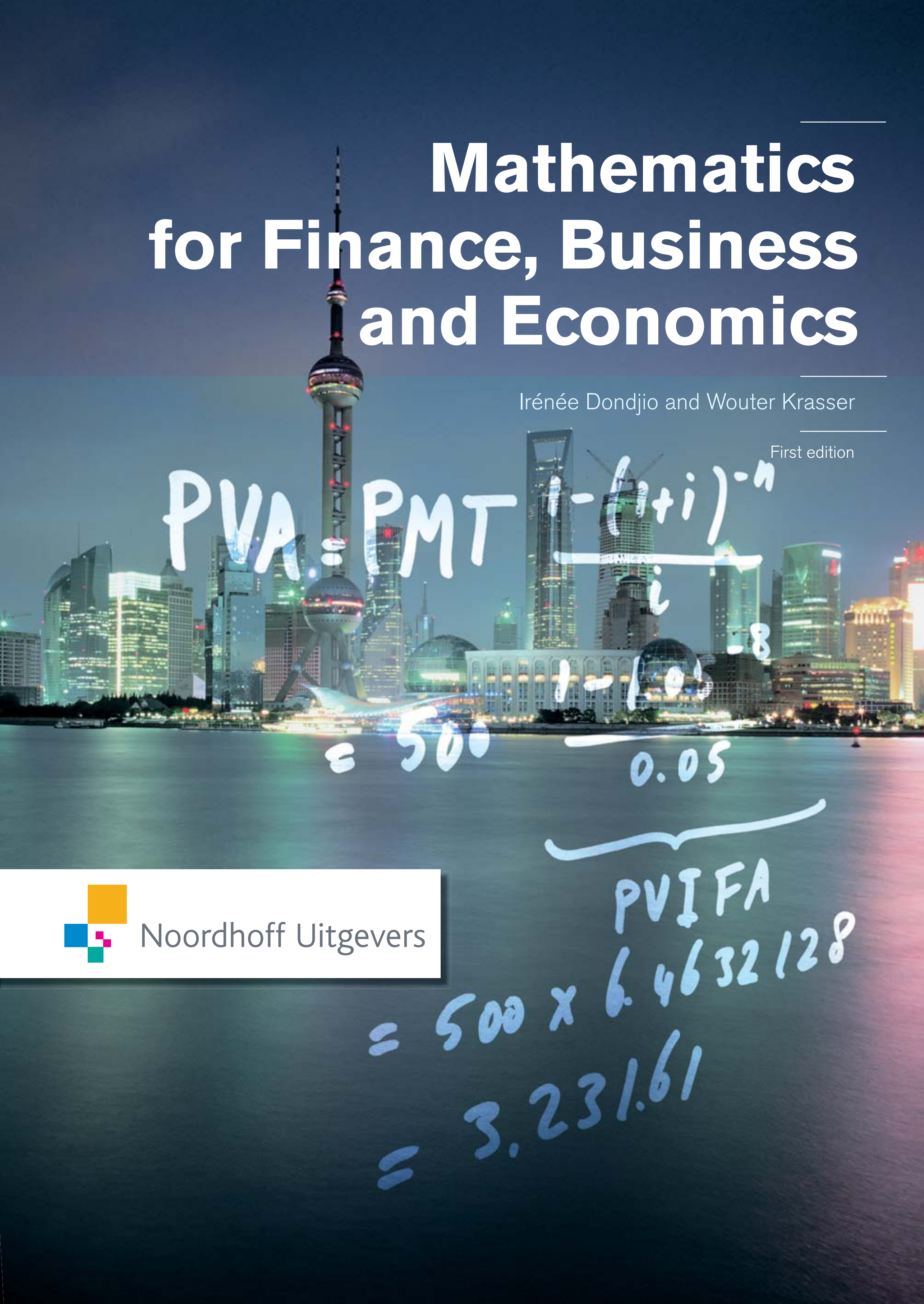

Mathematics for Finance, Business and Economics

Irénée Dondjio and Wouter Krasser

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Irénée Dondjio

Wouter Krasser

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Preface

Welcome to your new Mathematics book. Our desire to write this book was borne out of our experience of lecturing Mathematics, Economics and Marketing courses for five years at The Hague University of applied Science. The motivation arose when a couple of our students approached us and suggested writing a simplified Mathematics syllabus because the notes we gave in class were more comprehensive and easier to understand than the textbook.

It occurred to us that some students with an intermediate or advanced level of Mathematics were strongly motivated to tackle the course, whilst some with no background or a very poor knowledge of Mathematics tended to be daunted by the complex text, and this demotivated them. In order to improve students' knowledge of Mathematics, we decided to write a book that would be simple to understand, and make the subject more appealing and less off-putting to them.

This book is written informally, in order to be easy to read and understand. It's not just a lot of text, and then lots of exercises, such as you might find in traditional textbooks but a combination of explanations, explorations, and real-life applications of major concepts.

Acknowledgments
This book would not exist were it not for the efforts of The Hague University; we would like to express our appreciation for their unconditional support. In particular we would like to thank Ms. Akebe and Mr. Mugabi for sharing their expertise on the Mathematics of Finance, and Mr. Schumacher for making this book possible in the first place. Our gratitude also goes out to our colleagues from Saxion, Fontys, and the Hogeschool van Amsterdam for providing help and support in innumerable ways. Furthermore we would like to thank Donna Scott for editing the manuscript. And last, but not least we would like to acknowledge and extend our heartfelt gratitude to our family and friends for their encouragement and patient love which have enabled us to complete this handbook. We would especially like to express our gratitude to Etienne Nankeu.

Autumn 2012, The Hague
Irénée Dondjio, MBA.
Wouter Krasser, MSc.

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Introduction

For a long time, Mathematics has been seen as subject for highly skilled and specialized students. Business students are often not able to understand the benefits of Mathematics beyond simple everyday calculations.

Mathematics develops the imagination. It requires clear and logical thought. Mathematics is a science of patterns and structures that provides students with a uniquely powerful set of tools for understanding and solving real-life problems. Math studies non-tangible objects like numbers, circles and functions. However, we can profit greatly from this science because it has a big impact on our daily lives. For example, Math is essential in medicine, for analysing data on the causes of illness and the use of new drugs. Travel by aeroplane would not be possible without the Mathematics of airflow and control systems. Furthermore, Math is very useful in Finance, Economics and Accounting. For example, in exercise 6.19: suppose you have €1000 in a savings account with a yearly interest rate of 3%; how long would you have to wait until your money has doubled? Or exercise 5.10: can we build a model of the behaviour of customers in order to predict the market for the next few years? These and many other questions will be answered in this book.

All Business students need to master the basic concepts of Mathematics, as this is the key to understanding other courses, such as Economics, Finance, Statistics, and Accounting. Therefore, this book follows a logical order (from easy topics to challenging topics) so that students can stay focused.
Because this book is mainly dedicated to Business and Economics students, we have tried to relate it to subjects such as Economics, Marketing and Finance through real-life problems and situations.

In this book you will find theory, examples and exercises in each chapter. Solutions to the exercises can be found at the back of the book. The corresponding computations and more exercise material can be found on the website www.mathforfinance.noordhoff.nl.

The truncated icosahedron, aka the football

Source: Jan van de Craats, Faculteit FNWI UvA


Mc. 14,45

1 Elementary Mathematical Concepts and Operations

After studying this chapter you should be able to:

- add, subtract, multiply and divide positive and negative numbers
- understand the concept of a square root
- expand and evaluate an algebraic expression
- plot points on a graph

The aim of this chapter is to introduce the basic operations (addition, substraction, multiplication and division) used in algebra. The main focus will be on the properties of real numbers and the basic manipulation of algebraic expressions, collecting algebraic expressions with like terms.

Arithmetic is the ABC of Math. Addition, subtraction, multiplication, and division are the basics of Math and every Math operation known to humankind. In one way or another, every equation, graph, and many other things can be broken down into the ABCs of Math: the four basic operations. As people say, Math is a language, and addition, subtraction, multiplication, and division are its alphabet, along with the number line as well. The properties are basically proven ways to apply the mechanics of arithmetic to certain situations (source: www.writework.com).

1

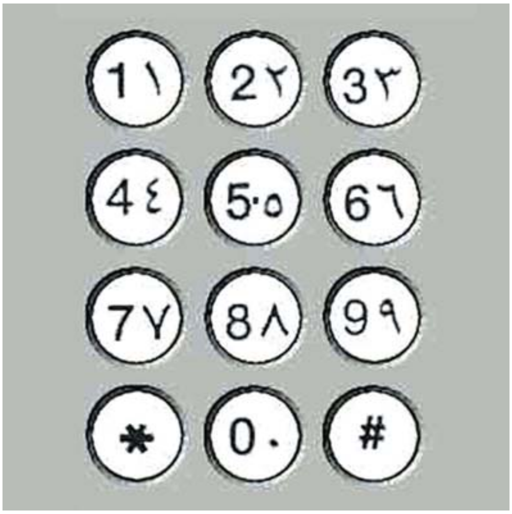
Is Mathematics only for a few kids?

Is Mathematics only for a few clever kids who are genetically disposed towards the subject? Many teachers believe, either explicitly or implicitly, that some children may be born with mathematical aptitudes or mathematics genes, and others are not. Some teachers even believe that children from certain groups (bases on factors such as gender, ethnicity and race) are blessed with superior mathematical ability. Some teachers feel there is not much that can be done to change or improve the innate ability of those unfortunate children who are inherently not good at mathematics.
The mathematical interests and knowledge young children bring to school may indeed differ, but the causes are more likely to be their varying experiences, rather than their biological endowment. While teachers should be aware of and sensitive to these differences, they should never lose sight of the fact that all children, regardless of their backgrounds and prior experiences, have the potential to learn Mathematics. In fact, the gaps in early Mathematics knowledge can be narrowed or even closed by good Mathematics curricula and teaching. Teachers should strive to hold high expectations and support for all children, without any ungrounded biases. When a teacher expects a child to succeed (or fail), the child tends to live up to that expectation.

Source: www.earlychildhoodaustralia.org

1.1 Simple Operations on Positive and Negative Numbers

FIGURE 1.1 An Arabic public telephone keypad

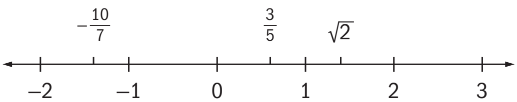


The digits 1, 2, etc. originate from Arabic numerals. Before adopting these symbols, Europeans used Roman numerals I, II, III, IV, V, etc. Negative numbers were not fully accepted until around 1800. In this book, we will use the decimal point and the thousand separator in our figures, like this:

$$12,345.67$$

The above means twelve thousand, three hundred forty-five and sixty-seven hundredths.

FIGURE 1.2 The number line illustrating the order of numbers



In Math there are positive and negative numbers, and the number 0, which is neither positive nor negative. We can think of a positive number as a credit and a negative number as a debt. The order of numbers is illustrated in figure 1.2, the picture of the number line, for example: $2 > 1$, $-2 < -1$, and $1 > -2$. Numbers can be added, subtracted and multiplied. All numbers can also be divided, but not by zero.
An example of the addition of positive and negative numbers goes like this: suppose you have a debt of €300. That means the balance of your bank account equals −€300. If you deposit €400 in your bank account, your new balance is $-300 + 400 = 100$ euros. Or suppose again you have a debt of €300, and your boyfriend has a debt of €200. So the balances of both of your bank accounts are −€300 and −€200 respectively. Now after your wedding you decide to join both of your bank accounts. Together you have a debt of €500, and the balance of your joint account equals $-300 + -200 = -500$ euros.

The multiplication and division of positive and negative numbers are illustrated in Table 1.1.

TABLE 1.1 Multiplication and division of positive and negative numbers

× or ÷	+	−
+	+	−
−	−	+

Example 1.1

Multiplication and Division of Positive and Negative Numbers Referring to Table 1.1 we see that:
$(+1) \times (+2) = 2$,
$(+1) \times (-2) = -2$,
$(-1) \times (+2) = -2$, and
$(-1) \times (-2) = 2$.

The same holds for division, i.e.
$\frac{+1}{+2} = +\frac{1}{2}$,

$\frac{+1}{-2} = -\frac{1}{2}$,

$\frac{-1}{+2} = -\frac{1}{2}$, and

$\frac{-1}{-2} = \frac{1}{2}$.

N.B. When there is no sign in front of the number, it is always positive. I.e. $3 = +3$.

EXTEND YOUR KNOWLEDGE
Do you know why $(-1)\times(-1)=1$?
The axioms of arithmetic state that for all numbers x, y, z we have:
a $x+-x=0$,
b $x\times 0=0$,
c $x\times(y+z)=x\times y+x\times z$ (distributivity), and
d $(-1)\times x=-x$.

Now:

$$\begin{aligned} 1+-1&=0 \text{ (by axiom a)},\\ (-1)\times(1+-1)&=(-1)\times 0 \text{ (by axiom b)},\\ (-1)\times 1+(-1)\times(-1)&=0 \text{ (by axiom c)},\\ -1+(-1)\times(-1)&=0 \text{ (by axiom d)},\\ \text{Hence } (-1)\times(-1)&=1. \end{aligned}$$

1.2 Fractions and Square Roots

A fraction is the ratio of two whole numbers, i.e.

$$\frac{6}{2}$$

is the ratio of 6 and 2. In this fraction 6 is called the *numerator*, and 2 the *denominator*. The ratio of 6 and 2 equals 3:

$$\frac{6}{2}=3,$$

for 6 is three times as big as 2:

$$6=3\times 2.$$

Using another example, we can also determine the ratio of $\frac{1}{2}$ and $\frac{1}{4}$:

$$\frac{\frac{1}{2}}{\frac{1}{4}}=2,$$

because $\frac{1}{2}$ is twice as big as $\frac{1}{4}$:

$$\frac{1}{2}=2\times\frac{1}{4}.$$

Two times a quarter equals a half may be demonstrated by: half a euro is twice as much as a quarter of a euro (50 cents is twice as much as 25 cents).

For calculations with fractions the following rules hold:

1

THEOREM 1.1

Calculation Rules for Fractions

Let a, b, c, and d be real numbers, with b and d unequal to zero. Then:

- The *addition* of two fractions with the *same* denominator is defined by:

$$\frac{a}{d}+\frac{c}{d}=\frac{a+c}{d} \qquad (1.1)$$

- The *addition* of two fractions with *different* denominators is defined by:

$$\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd} \qquad (1.2)$$

- The *multiplication* of two fractions is defined by:

$$\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd} \qquad (1.3)$$

- The *division* of two fractions is defined by:

$$\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a}{b}\times\frac{d}{c}=\frac{ad}{bc} \qquad (1.4)$$

Equation (1.4) shows that dividing by a fraction is equivalent to multiplying by its inverse (the inverse of a number $x \neq 0$ is $\frac{1}{x}$). Note that we have used the notational convention ab for $a \times b$.

Here is an example of the application of equation (1.1): Suppose you have $\frac{1}{5}$ of a euro, 20 cents. You add $\frac{2}{5}$ of a euro, 40 cents. The result is 60 cents, $\frac{3}{5}$ of a euro:

$$\frac{1}{5}+\frac{2}{5}=\frac{1+2}{5}=\frac{3}{5}$$

Here is an example of the application of equation (1.2): Suppose you have $\frac{1}{2}$ of a euro, 50 cents. You add $\frac{3}{5}$ of a euro, 60 cents. The result is 110 cents, 1.1 euros:

$$\frac{1}{2}+\frac{3}{5}$$

Since two fractions with a different denominator cannot be added immediately, we use equation (1.2):

$$\frac{1}{2}+\frac{3}{5}=\frac{1\times5}{2\times5}+\frac{3\times2}{2\times5}=\frac{1\times5+3\times2}{10}=\frac{11}{10}=1.1$$

Here is an example of the application of equation (1.3):

$$\frac{1}{3}\times\frac{2}{5}=\frac{1\times2}{3\times5}=\frac{2}{15}$$

Here is an example of the application of equation (1.4):

$$\frac{\frac{1}{2}}{\frac{3}{4}}=\frac{1}{2}\times\frac{4}{3}=\frac{1\times4}{2\times3}=\frac{4}{6}=\frac{2}{3}$$

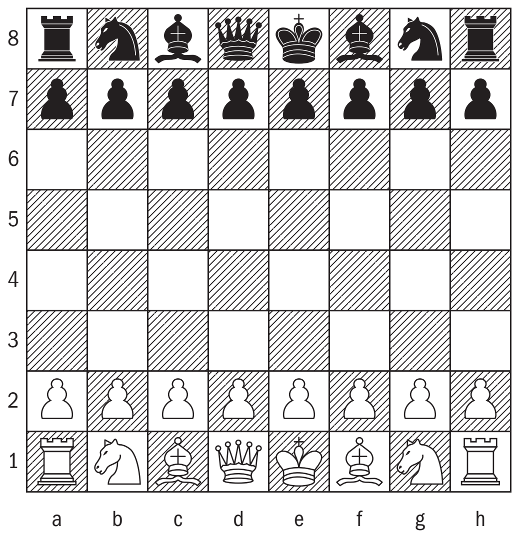
We will now turn to the definition of the square and the root of a number. The *square of a number* is that of a number multiplied by itself, and the *square root* answers the question ‘Which number was multiplied by itself in order to get this new number?’. A square root is useful for solving an equation like $x^2=64$ (see figure 1.3).

Definition 1.1 Square
Let x be any number. The square of x, x^2 is defined by

$$x^2=x\times x$$

For example: $5^2=5\times5=25$ and $(-3)^2=(-3)\times(-3)=9$.

FIGURE 1.3 The square of 8 equals 64 because $8\times8=64$



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Definition 1.2 Square root
Let x be a non-negative number, then the square root of x,

$$y = \sqrt{x},$$

is that number $y \geq 0$ which, when squared, equals x, i.e.:

$$y^2 = (\sqrt{x})^2$$

Example 1.2

The square root of 9 equals 3, because $3^2 = 9$. Note also that $(-3)^2 = 9$, so you might think that -3 would also be the square root of 9, however, the definition of a square root states that only the non-negative value applies. Note that negative numbers do not have a square root. For example: $\sqrt{-9}$ does not exist because there is no number that, when squared, equals -9.

THEOREM 1.2

Calculation Rules for the Square Root

Let a, $b \geq 0$, $c > 0$, and d be any number. Then consider the following rules:

$$\sqrt{ab} = \sqrt{a}\sqrt{b} \tag{1.5}$$

$$\sqrt{\frac{a}{c}} = \frac{\sqrt{a}}{\sqrt{c}} \tag{1.6}$$

$$\sqrt{d^2} = |d| \tag{1.7}$$

$$(\sqrt{a})^2 = a \tag{1.8}$$

Where $|d|$ denotes the absolute value of d, as defined by:

$$|d| = \begin{cases} -d \text{ for } d < 0 \\ 0 \text{ for } d = 0 \\ d \text{ for } d > 0 \end{cases} \tag{1.9}$$

For example: $|-2| = 2$ and $|2| = 2$. Basically, to arrive at the absolute value of a number expressed with a minus sign, you just need to remove that minus sign.

Example 1.3

An example of equation (1.5) is:
$\sqrt{8} = \sqrt{4 \times 2} = \sqrt{4}\sqrt{2} = 2\sqrt{2}$

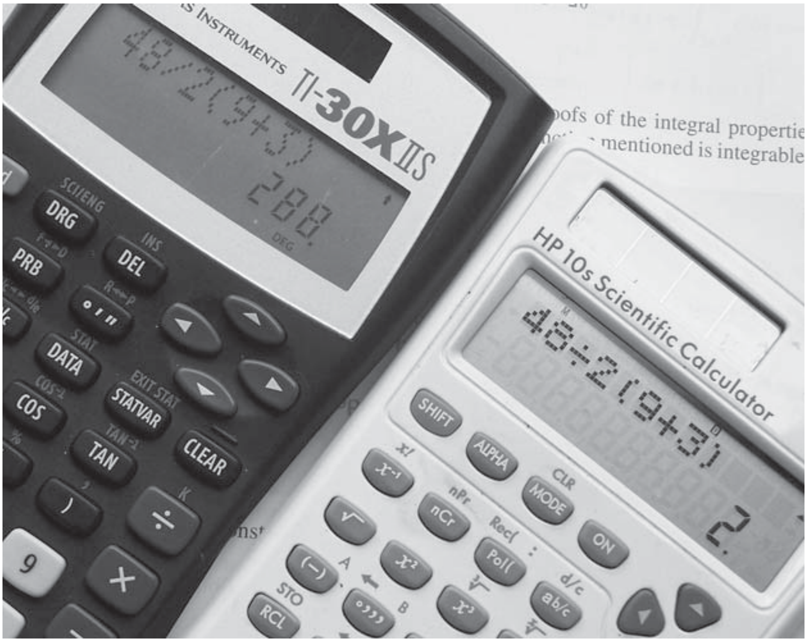
Example 1.4

An example of equation (1.7) and the absolute value (1.9) is:
$\sqrt{(-4)^2} = |-4| = 4$

1.3 BEDMAS

Just as it matters in which order you put on your shoes and socks, it matters in which order you add and multiply. The order in which mathematical operations are executed is often referred to as 'BEDMAS' (see figure 1.4).

FIGURE 1.4 Which of the calculators is right?



'BEDMAS' is an acronym that stands for:

- **B** brackets
- **E** exponents (means powers, squares and roots)
- **DM** divide and multiply, from left to right
- **AS** add and subtract, from left to right

Example 1.5

Let us calculate:

$$\frac{(3+6)-8\times 3}{24+6^2}$$

Following 'BEDMAS' we need to start with the **B**rackets:

$$(3+6)-8\times 3\div 24+6^2$$
$$=9-8\times 3\div 24+6^2$$

There is an **E**xponent, 6^2, which has to be done now:

$$9-8\times 3\div 24+6^2$$
$$=9-8\times 3\div 24+36$$

We continue with **D**ivision and **M**ultiplication from left to right:

$$\begin{aligned} & 9-8\times 3\div 24+36 \\ =\ & 9-24\div 24+36 \\ =\ & 9-1+36 \end{aligned}$$

and we finish with **A**ddition and **S**ubtraction from left to right:

$$\begin{aligned} & 9-1+36 \\ =\ & 8+36 \\ =\ & 44 \end{aligned}$$

1.4 Algebraic Expressions

Definition 1.3 Algebraic expression
An algebraic expression is made up of the signs, or numbers, and letters, or symbols of algebra. It is also composed of terms which can be variable or constant.

$$\overbrace{\underbrace{5}_{\text{coefficient}}\ \underbrace{x}_{\text{variable}}}^{\text{term}} - \overbrace{\underbrace{3}_{\text{constant}}}^{\text{term}}$$

Definition 1.4 Variable
Variables are unknown values that may change within the scope of a given problem or set of operations. Variables are represented by letters most of the time, and the ones often used are: x, y, and z. However any other letter could be used.

Definition 1.5 Constant
A constant is a fixed value that does not change, like 1, 2, 3 etc.

Example 1.6

Addition and Subtraction of Algebraic Expressions
How can we simplify an expression such as
$2x+4y-25x+30y-45xy+25yx$?

First we collect like terms; there are 3 different sorts, x, y, and xy:

x	y	xy
$2x-25x$	$4y+30y$	$-45xy+25yx$
$=-23x$	$=34y$	$=-20xy$
		Note that $xy=yx$.

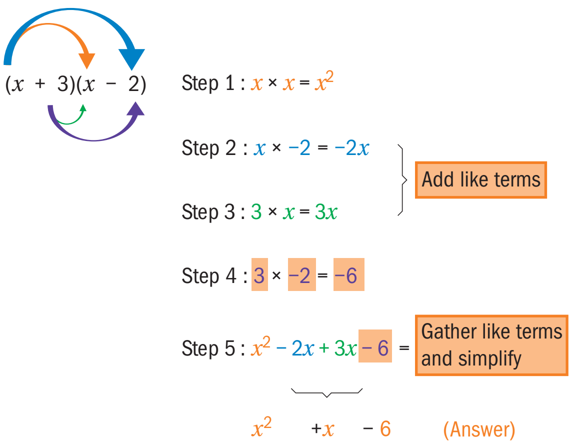
After adding all the terms we get $-23x+34y-20xy$.

1.5 Expanding Brackets

Expanding brackets, otherwise known as evaluating an expression, involves removing the brackets in order to simplify the expression down to a single numerical value. We can expand the brackets of the following expression:

$$(x+3)(x-2)$$

FIGURE 1.5 The banana method



A convenient way of expanding these brackets is by using the *banana method*, see figure 1.5:

$$(x+3)(x-2)$$

$$\begin{aligned}(x+3)(x-2) &= x^2-2x+3x-6\\ &= x^2+x-6\end{aligned}$$

Does this make any sense? Let us check the following example. We already know that:

$$(1+2)(3+4)=(3)(7)=3\times 7=21 \qquad (1.10)$$

Expanding the brackets first yields:

$$\begin{aligned}(1+2)(3+4) &= 3+4+6+8\\ &= 21\end{aligned}$$

Which is the same answer as in (1.10).

1.6 Factorizing Algebraic Expressions

Factorizing an algebraic expression is the opposite of expanding. You start with a sum or difference of terms and finish up with a product.

For example, by factorizing the following expression:

$$ab + ac$$

You get

$$a(b + c)$$

We have used the *common factor method* in the example: all terms have a as a factor, so a is the common factor. We should now use the brackets to write down the common factor outside the brackets as follows:

$$a(\quad)$$

To find out what goes inside the brackets, divide the original terms by the common factor(s).

For the example, divide the original terms by a:

$$\frac{ab}{a} = b$$

$$\frac{ac}{a} = c$$

The last step is to write these new expressions inside the brackets, like this:

$$a(b + c)$$

Example 1.7

Factorize $2yx + 4x$
$2yx + 4x$
$= 2xy + 2x \times 2$
$= 2x(y + 2)$

1.7 The Cartesian Plane

The Cartesian plane was named after the famous French philosopher and mathematician Rene Descartes (figure 1.6). When two perpendicular number lines intersect, a Cartesian plane is formed.

Rene Descartes, a French philosopher, viewed the world with a cold analytical logic. He saw all physical bodies, including the human body, as machines operated by mechanical principles. His philosophy was derived from the austere logic of 'cogito ergo sum' meaning: I think therefore I am.
In Mathematics, Descartes's chief contribution was in analytical geometry. Descartes's portrait is quadrisected by the axes of his great advance in analytical geometry: the Cartesian Plane. It enabled an algebraic representation of geometry (source: http://mathematicianspictures.com).

FIGURE 1.6 René Descartes (1596–1650)

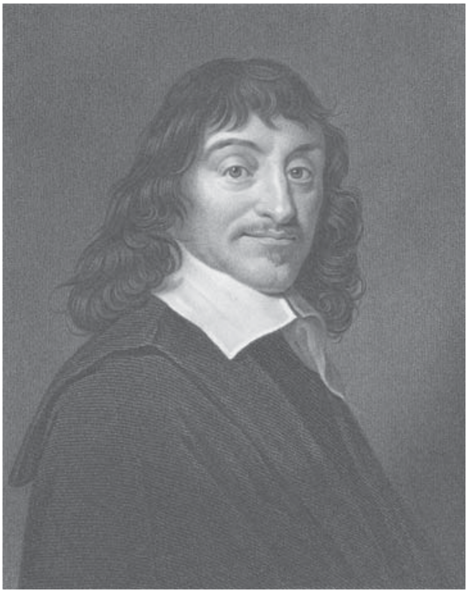
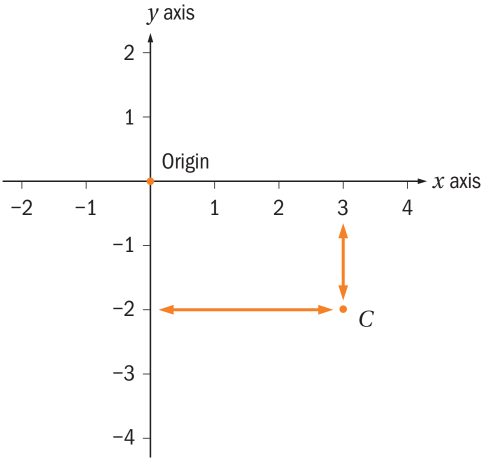


FIGURE 1.7 The coordinate plane



The Cartesian Plane is a plane with a rectangular coordinate system that associates each point in the plane with a pair of numbers. A coordinate plane has two axes: the x-axis (the horizontal line), and the y-axis (the vertical line). These two axes originate in the origin O with coordinates (0, 0). In figure 1.7 point C has coordinates $(3, -2)$. 3 is the distance of this point from the origin in the direction of the x-axis, and -2 is the distance of this point from the origin in the direction of the y-axis, hence the line goes downwards because -2 is negative.

1

Summary

- Multiplication and division of positive and negative numbers are illustrated in this table:

× or ÷	+	−
+	+	−
−	−	+

- Rules for calculations with fractions are:
  - $\frac{a}{d}+\frac{c}{d}=\frac{a+c}{d}$
  - $\frac{a}{b}+\frac{c}{d}=\frac{ad}{bd}+\frac{bc}{bd}=\frac{ad+bc}{bd}$
  - $\frac{a}{d}\times\frac{c}{d}=\frac{ac}{bd}$
  - $\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a}{b}\times\frac{d}{c}=\frac{ad}{bc}$
- $x^2=x\times x$
- $y=\sqrt{x}$ shows that non negative number y satisfies $y^2=x$
- Rules for calculations with square roots are:
  - $\sqrt{ab}=\sqrt{a}\sqrt{b}$
  - $\sqrt{\frac{a}{c}}=\frac{\sqrt{a}}{\sqrt{c}}$
  - $(\sqrt{a})^2=a$
  - The square root of a negative number does not exist.
  - $\sqrt{d^2}=|d|$,

where $|d|$ denotes the absolute value of d, as defined by

$$|d|=\begin{cases}-d \text{ for } d<0\\ 0 \text{ for } d=0\\ d \text{ for } d>0\end{cases}$$

- BEDMAS tells you the order to follow: brackets, exponents, division and multiplication (from left to right), addition and subtraction (from left to right)
- Expanding brackets:

$$(a+b)(c+d)=ac+ad+bc+bd$$

- What is the difference between variable, constant and coefficient?

Exercises

1

Complete these exercises without using a calculator:

1.1 You borrowed €1,000 from the bank last month to buy your books, and this month you paid back €800. But you realize that you need a notebook, so you borrow another €500 to buy one. What is the total amount that you owe the bank now?

1.2 Last week the balance of your bank account was €600 in debit and this week you withdrew €300. How much is your account now in debit by?

1.3 Determine the validity of the following statements. Are they true or false?

a $-3 \leq 4$
b $8 < 3$
c $-8 < -3$
d $0.7 \geq -4$
e $-5.3 > -7$
f $\frac{3}{4} < \frac{6}{8}$
g $-\frac{3}{4} \leq -\frac{6}{8}$
h $1 + 2 = 2 + 1$
i $1 - 2 = 2 - 1$

1.4 Calculate

a $\frac{1}{3} + \frac{1}{3}$
b $\frac{1}{3} + \frac{2}{3}$
c $\frac{1}{3} + \frac{1}{6}$
d $\frac{1}{2} - \frac{1}{4}$
e $\frac{1}{2} + \frac{1}{3}$
f $\frac{1}{2} - \frac{2}{3}$
g $\frac{1}{3} + \frac{3}{2}$
h $\frac{5}{7} - \frac{7}{5}$

1.5 Calculate

a $\frac{1}{2}\times\frac{1}{2}$

b $\frac{1}{3}\times\frac{2}{1}$

c $\frac{1}{3}\times 2$

d $\frac{2}{3}\times\frac{-1}{2}$

e $\frac{8}{-7}\times\frac{3}{5}$

f $\frac{7}{8}\times\frac{-5}{3}$

g $\frac{3}{5}\times\frac{5}{2}$

h $\frac{5}{7}\times\frac{7}{5}$

1.6 Calculate

a $3+(6-2)$

b $5-(8-7)$

c $15-(4-2)$

d $(17-1)-4$

e $23-(4-2)$

f $5+(14-7)-3$

g $4+(71-1)+1$

h $16-(8-7)-5$

1.7 Calculate

a $\frac{5}{4}\times\frac{7}{8}$

b $\frac{18}{7}+\frac{3}{8}$

c $-\frac{5}{7}+\frac{7}{5}+\frac{3}{2}$

d $-\left(\frac{7}{5}+\frac{5}{9}\right)+\left(-\frac{3}{9}+\left(-\frac{14}{5}\right)\right)$

e $-\left(\frac{10}{5}-\frac{5}{8}\right)-\left(-\frac{3}{9}-\left(-\frac{12}{5}\right)\right)$

f $\frac{\frac{1}{2}}{3}$, hint: $3=\frac{3}{1}$

g $\frac{1}{\frac{2}{3}}$

h $\frac{\frac{1}{4}}{\frac{2}{3}}$

i $\frac{1}{\frac{1}{2}}$

1

1

j $\dfrac{\frac{1}{1}}{2}$

k $\dfrac{\frac{1}{2}}{\frac{1}{3}}$

l $\dfrac{\frac{1}{3}}{\frac{1}{2}}$

1.8 Calculate

a $(1-2)-3$
b $1-(2-3)$
c $2\times3+4$
d $2\times(3+4)$
e $10-5+8$
f $10-(5+8)$
g $10\div2+3$
h $10\div(2+3)$

1.9 Evaluate and simplify

a $\sqrt{9}$
b $\sqrt{27}$
c $\sqrt{4\times9}$
d $\sqrt{(-9)^2}$
e $\sqrt{\dfrac{9}{16}}$
f $\sqrt{\dfrac{27}{8}}$
g $(\sqrt{3})^2$
h $\sqrt{3^2}$

1.10 Calculate

a $(7+6)^2\div13+1$
b $(1+2)^3+4\div5-6\times7$
c $(3+5-7)\times6\div2+3^2$
d $(3+5)-7\times6\div2+3^2$
e $1\div\dfrac{2}{3}$
f $\dfrac{1}{2}\div3$
g $\dfrac{1}{2}+\dfrac{3}{4}$
h $\dfrac{1+2}{3+4}$
i $1+2\div3+4$
j $\dfrac{\frac{1}{2}}{3}+4$

1.11 Referring to figure 1.4, which calculator is right according to BEDMAS?

1.12 Calculate

a $(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})$

b $2\sqrt{6}-\sqrt{2}\sqrt{3}$

c $\dfrac{\sqrt{4}}{2}$

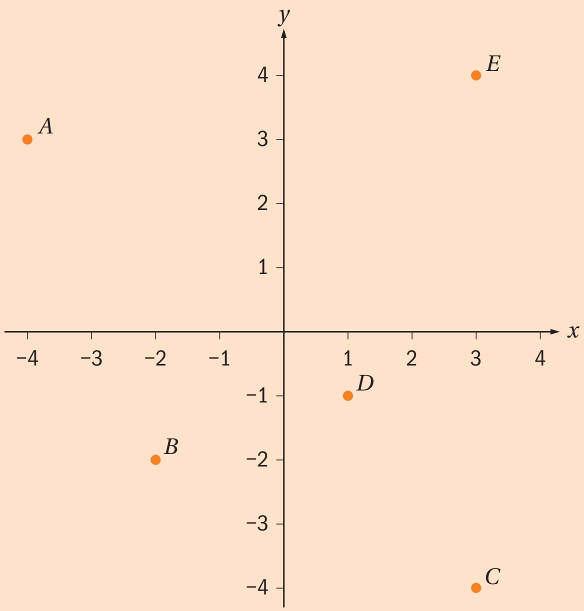
d $\sqrt{\dfrac{4}{2}}$

e $\sqrt{4+5}$

f $\sqrt{4}+5$

g $\sqrt{3\times 3+20-8\div 2}$

h $\sqrt{3\times 3}+\sqrt{20-8\div 2}$



1.13 Use this coordinate plane to answer the following questions:

a Which point has coordinates $(3, -4)$?

b What are the coordinates of the other points $A, B, \ldots, E$?

c Is there a point plotted at coordinates $(1, -1)$?

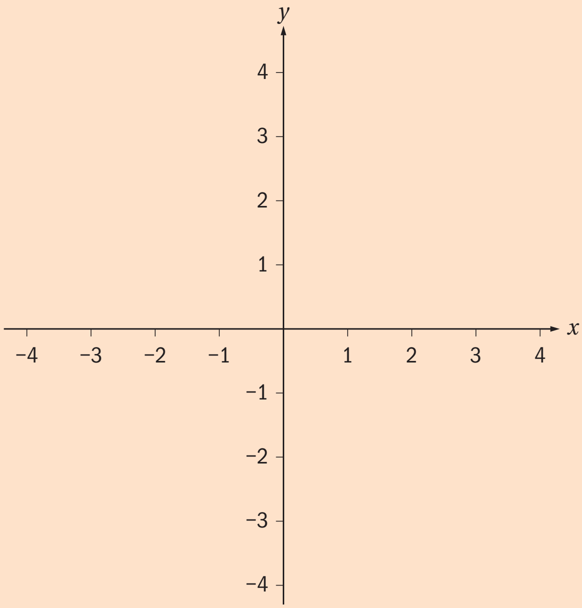
1.14 Plot the following points in this coordinate plane.

$A=(4, 0)$ $B=(1, 2)$ $C=(2, 1)$ $D=(-3, 3)$

$E=(-4, 0)$ $F=(-1, -2)$ $G=(2, -1)$ $H=(0; 0.7)$

$I=(-0.8; 1)$ $J=\left(-3\frac{1}{2}, 1\right)$ $K=\left(\frac{5}{2}, \frac{7}{2}\right)$ $L=\left(-\frac{18}{17}, \frac{22}{7}\right)$

1



1.15 Simplify the following expressions by collecting like terms:

a $8x-4x+7y-5x+x+3y-10y$
b $5u-7v+6uv-5vu+8v-4u$

1.16 Expand the following expressions and simplify the result.

a $7(x+2y)+8(2x-y)$
b $10(p+q)-3(p-q)$
c $4(x+6)+5(2x+6)$
d $-4(-2x+5)+5(-3x+6)$

1.17 Simplify the following expressions by expanding the brackets and adding like terms: [see fig 1.4]

a $(x+7)(x-4)$
b $(x-y)(x+y)$
c $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})$
d $(x+y)^2$
e $(2b+6)(b-7)$
f $(2x+5y)+(3x-2y)$
g $(7.5x^2-x-4)+\left(\frac{1}{2}x^2-2x-3\right)+\frac{4}{2}x^2$

1.18 Show that
$\sqrt{2}+\sqrt{3}=\sqrt{5+2\sqrt{6}}$

1.19 Factorize the following expressions:

a $3x+3$
b $(x+1)(x+5)+7(x+1)$
c $(x+1)(3x+4)+(x+1)(x-3)$
d $(x+1)(4x+9)-5(x+1)$