## Chapter 1

## BASIC CONCEPTS

### 1.1 Introduction

This course book deals with performance prediction of aircraft. By performance we understand certain extremes of quantities that are related to the translational motion of the vehicle, such as: rate of climb, flight regime, takeoff and landing distance, range and endurance, turning rate, etc.
In this book the subject matter is limited to that class of aircraft known as airplanes. An airplane may be defined as a mechanically driven fixed-wing aircraft, heavier than air, which is supported by the reaction forces caused by the airflow against the surface of its body. Moreover, the attention is devoted to the examination of the performance of existing airplanes so that - in principle - pertinent airplane data are available. This means that the problem of designing an airplane that meets specified performance requirements, will not be discussed.
As a simplification, performance will be represented by the translational motion of the airplane as a response to the external forces acting on the center of mass of the airplane. The prerequisite for this treatment is the assunption that the airplane is regarded a rigid body.
Another important idealization may be the assumption of an airplane flying over an Earth that is considered to be nonrotating and flat. The different approximations will be discussed in some detail in subsequent sections of this chapter.

### 1.2 The airplane is regarded a rigid body

In this book we shall limit our analyses to rigid airplanes. In the case of rigidity, the motion of an airplane can be divided into a translational and a rotational motion.


Figure 1.1 Division of airplane motion


Figure 1.2 Determination of center of gravity

As illustrated by Figure 1.1, a rigid airplane has six degrees of freedom; the three components of the linear velocity and the three components of the angular velocity, acting along and about the $X, Y$ and $Z$ axes, respectively, where the origin of the axis system coincides with the center of mass of the airplane (see Appendix A).

The displacement of the airplane can be determined by treating the airplane as a point mass located at the center of mass, customarily referred to as the center of gravity (abbreviation: c.g.).
The rotation of the airplane depends on the moments about the center of gravity. The effects the moments have on the rotation of the airplane are studied in the field of aeronautics, called stability and control. The subjects stability and control concern the abilities to maintain and to change prescribed flight conditions, respectively.
Since throughout this book the emphasis is on the computation of airplane performance, we can limit our considerations to the effects that the application of the external forces and moments have on the displacement of the center of gravity of the airplane.
According to its definition, the center of gravity of an airplane is the point through which the resultant of the partial weights acts, independent of the attitude of the airplane.
The location of the center of gravity in longitudinal direction can be found by measuring the reaction forces $N_{1}$ and $N_{2}$ in the ground-based situation (Figure 1.2).

The sum of the loads at each wheel is equal to the weight of the airplane:

$$
W=N_{1}+N_{2} .
$$

The sum of the moments of the loads at each wheel equals the weight multiplied by the distance between the center of gravity and the reference line:

$$
N_{1} X_{1}+N_{2} X_{2}=W X_{z}
$$

From the latter equality we obtain

$$
\begin{equation*}
X_{z}=\frac{N_{1} X_{1}+N_{2} X_{2}}{W} . \tag{1.1}
\end{equation*}
$$

In order to determine the location of the center of gravity in vertical direction, weighing must be executed at inclined airplane positions.
To ensure save and convenient operation, every pilot has to be aware of the airplane weight, as well as the way this weight is distributed in the airplane, in order to make sure that allowable weight and approved center of gravity limits are not exceeded.
A typical light airplane loading graph and center of gravity moment envelope are sketched in Figures 1.3 and 1.4.
For a properly loaded airplane the actual weight and moment values must fall within the lines indicating forward and aft center of gravity.


Figure 1.3 Loading graph


Figure 1.4 Center of gravity moment envelope

### 1.3 Application of Newton's law of motion with respect to an axis system attached to the Earth

The translational motion of a rigid body with constant mass is described by Newton's second law of motion :

$$
\begin{equation*}
F=M a \tag{1.2}
\end{equation*}
$$

where $F$ is the vector sum of all external forces acting on the body, $M$ is its mass, and $a$ is the absolute acceleration. Equation (1.2) must be written down with respect to an inertial frame of reference, that is to say, an axis system in a state of complete rest, or any coordinate system which translates with uniform velocity relative to the frame at rest.
According to the analysis in Appendix A, we can apply Equation (1.2) in a coordinate system attached to the Earth if two apparent forces are added to the force $F$,

$$
\begin{equation*}
F-M \omega_{e} \times\left[\omega_{e} \times(R+h)\right]-M\left(2 \omega_{e} \times V\right)=M a_{r}, \tag{1.3}
\end{equation*}
$$



Figure 1.5 Forces due to the rotation of the Earth.
where $\omega_{e}$ is the Earth's angular velocity (about $7.29 \times 10^{-5}$ radians per second), $R$ is the Earth's radius vector, $h$ is the height above the surface of the Earth, $V$ is the velocity of the body with respect to the Earth, and $a_{r}$ is the acceleration relative to the Earth.
The second term of the left-hand side of Equation (1.3) is a centrifugal force,

$$
\begin{equation*}
F_{t}=-M \omega_{e} \times\left[\omega_{e} \times(R+h)\right]=-M a_{t}, \tag{1.4}
\end{equation*}
$$

where $a_{t}$ is a centripetal acceleration. The last term is a Coriolis force,

$$
\begin{equation*}
F_{c}=-M\left(2 \omega_{e} \times V\right)=-M a_{c}, \tag{1.5}
\end{equation*}
$$

where $a_{c}$ is the Coriolis acceleration.
In deriving Equation (1.3), the assumption is made that the Earth translates with a constant velocity along a straight line. This idealization of the Earth's translational motion may be correct since our performance analyses normally deal with small time intervals, that is, small with respect to the period of revolution of the Earth around the Sun.
By expressing the Equations (1.4) and (1.5) in trigonometric form, we obtain for the magnitude of the centrifugal force:

$$
\begin{equation*}
F_{t}=M \omega_{e}^{2}(R+h) \cos \theta \tag{1.6}
\end{equation*}
$$

and for the Coriolis force:

$$
\begin{equation*}
F_{c}=M 2 \omega_{e} V \sin \phi \tag{1.7}
\end{equation*}
$$

In the latter expressions the angle $\theta$ is latitude, positive in the Northern Hemisphere and negative in the Southern. The angle $\phi$ defines the direction of the velocity relative to the Polar axis. The forces and geometry used in Equations (1.6) and (1.7) are depicted in Figure 1.5. It is interesting to note that the radius to the North Pole is somewhat larger than the radius to the South Pole. This deviation from the sphere is indicated as the pear shape of the Earth.


Figure 1.6 Coriolis forces

The centrifugal force is directed perpendicular to the Earth's Polar axis and points out from the Earth along a line intersecting the axis of rotation. At a position in the equator plane $\left(\theta=0^{\circ}\right)$ and near the Earth's surface we obtain for the magnitude of the centripetal acceleration, using the approximation that the Earth may be regarded a sphere with a radius $R_{e}=6371 \mathrm{~km}$

$$
a_{t}=\omega_{e}^{2} R_{e}=\left(7.29 \times 10^{-5}\right)^{2} \times 6371 \times 10^{3}=0.034 \mathrm{~m} / \mathrm{s}^{2}
$$

Equation (1.7) shows that at a given velocity, the Coriolis acceleration, $a_{c}$, has its maximum value when the velocity is directed perpendicular to the polar axis ( $\phi=90^{\circ}$ ).
To illustrate the Coriolis force in more detail, the effect of a vertical velocity, a northward velocity, and an eastward velocity is considered successively in Figure 1.6, where the body is in a point on the Earth's surface. It follows from Equation (1.5) that a body moving vertically upward appears to an axis system referenced to the rotating Earth to be forced to the west by

$$
F_{c}=M 2 \omega_{e} V \sin (90-\theta)
$$

When the body has a northward velocity, the Coriolis force becomes

$$
F_{c}=M 2 \omega_{e} V \sin \theta
$$

In this case the body is subject to an eastward force.
Figure 1.6, finally, shows that a body with an eastward velocity appears to be forced outward from the Earth. It is seen from Equation (1.7) that now the Coriolis force is not dependent on latitude:

$$
\begin{equation*}
F_{c}=M 2 \omega_{e} V . \tag{1.8}
\end{equation*}
$$

This force can be resolved into a component directed upward along the radius vector of the Earth, and a southward component.


Figure 1.7 Typical flight velocities and altitudes.

In order to provide an idea of the degree of importance of the Coriolis acceleration, assume a body with a velocity of $2000 \mathrm{~km} / \mathrm{h}$ to the east. Then,

$$
a_{c}=2 \omega_{e} V=2 \times 7.29 \times 10^{-5} \frac{2000}{3.6}=0.081 \mathrm{~m} / \mathrm{s}^{2}
$$

Anticipating the discussion on gravitation in the following section, it can be noticed here that in comparison with the acceleration of gravity $\left(=9.81 \mathrm{~m} / \mathrm{s}^{2}\right.$ at the Earth's surface) the centripetal accelerations as well as the Coriolis accelerations are very small.
From the numerical examples given before it will also be clear that the effects of rotation of the Earth on the motion of a body may only become of interest in the study of high-altitude and high-velocity vehicles. This means that these effects are negligibie for most airplane operations, which are executed at lower altitudes and at relatively low airspeeds. The latter conditions are evident from Figure 1.7, where are shown flight altitudes and airspeeds for typical airplane types.
Especially the layer below 20 km is an important region to aeronautics since most airplane operations are executed in this atmospheric shell.

### 1.4 Gravitation

Newton's law of gravitation states that any two particles attract one another with a force of magnitude :

$$
\begin{equation*}
F=\frac{\mu M_{1} M_{2}}{R^{2}} \tag{1.9}
\end{equation*}
$$



Figure 1.8 Components of gravity force
where $M_{1}, M_{2}$ are the masses of the particles, $R$ is the distance between them and $\mu$ is a proportionality factor, known as the universal gravitational constant. The force $F$ acts along the line joining the particles. Accordingly, if $M$ is the mass of a particle outside the Earth, and $M_{e}$ the mass of the Earth, the gravitational force $F_{g}$ on the particle is given by (Figure 1.8)

$$
\begin{equation*}
F_{g}=\frac{\mu M_{e} M}{\left(R_{e}+h\right)^{2}} \tag{1.10}
\end{equation*}
$$

This equation says that the gravitational force due to the Earth is the same as if all mass $M_{e}$ were concentrated at the center of the Earth. To derive Equation (1.10) the assumption must be made that the Earth can be considered a sphere (mean radius $R_{e}=6371 \mathrm{~km}$ ), of which the density is a function of the distance to the center only.
As shown in Figure 1.8, the gravity force or weight $W$ of a body is actually the vector sum of the gravitational force $F_{g}$ and the centrifugal force $F_{t}$ due to the rotation of the Earth about its Polar axis. Therefore, the gravity force does not point exactly to the center of the Earth.
The centrifugal force in Figure 1.8 results from the choice of an earthbound rotating frame of reference and is given by Equation (1.6), repeated below,

$$
\begin{equation*}
F_{t}=M \omega_{e}^{2}\left(R_{e}+h\right) \cos \theta \tag{1.11}
\end{equation*}
$$

The gravity force per unit mass is the acceleration of gravity, $g=W / M$.
The sea-level value of $g$ may be given by, from Equations (1.10) and (1.11), and Figure 1.8,

$$
\begin{equation*}
g=\frac{\mu M_{e}}{R_{e}^{2}}-\omega_{e}^{2} R_{e} \cos ^{2} \theta \tag{1.12}
\end{equation*}
$$

At the Equator the centrifugal force is a maximum. There we get for the acceleration of gravity at the Earth's surface, using $\mu=6.67 \times 10^{-11} \mathrm{~m}^{3} \mathrm{~kg}^{-1} \mathrm{~s}^{-2}, M_{e}=$ $5.98 \times 10^{24} \mathrm{~kg}, R_{e}=6.371 \times 10^{6} \mathrm{~m}$, and $\omega_{e}=7.29 \times 10^{-5} \mathrm{~s}^{-1}$ :

$$
g=\frac{\mu M_{e}}{R_{e}^{2}}-\omega_{e}^{2} R_{e}=9.827-0.034=9.793 \mathrm{~m} / \mathrm{s}^{2}
$$

Because of the variation of the centrifugal force with latitude, the above valuc of $g$ increases gradually to $9.827 \mathrm{~m} / \mathrm{s}^{2}$ at the Poles $\left(\theta=90^{\circ}\right)$.
At $45^{\circ}$ geographic latitude the sea-level acceleration of gravity, denoted $g_{0}$, becomes

$$
g_{0}=\frac{\mu M_{e}}{R_{e}^{2}}-\omega_{e}^{2} R_{e} \cos ^{2} \theta=9.827-0.034 \times 0.5=9.810 \mathrm{~m} / \mathrm{s}^{2}
$$

At this point, it is worthy to note that in particular for the International Standard Atmosphere (see Chapter 2) the acceleration of gravity $g_{0}$ is used and taken as $9.80665 \mathrm{~m} / \mathrm{s}^{2}$.
In our applications, mostly, it is possible to ignore the effect of the centrifugal force when considering the variation of $g$ with height. Then, it follows from Equation (1.10) that the acceleration of gravity varies inversely as the square of the distance from the center of the Earth

$$
\begin{align*}
& g=\frac{\mu M_{e}}{\left(R_{e}+h\right)^{2}}, \quad \text { and }  \tag{1.13}\\
& \frac{g}{g_{0}}=\frac{R_{e}^{2}}{\left(R_{e}+h\right)^{2}}=\left(1+\frac{h}{R_{e}}\right)^{-2} . \tag{1.14}
\end{align*}
$$

Using the first two terms of the binomial expansion we get

$$
\begin{equation*}
\frac{g}{g_{0}}=1-2 \frac{h}{R_{e}} \tag{1.15}
\end{equation*}
$$

The latter expression may show that even at the maximum altitudes suitable for atmospheric flight ( $h=60-80 \mathrm{~km}$ ), there is only a slight difference between $g$ and $g_{0}$. But certainly at the heights encountered during normal operations (less than 20 km ) the actual value of $g$ is very near to its standard sea-level value $\left(g / g_{0}=\right.$ 0.993 at $h=20 \mathrm{~km}$ ).

### 1.5 The effect of curvature of the Earth

Considered is a body moving at constant speed in a circular orbit of radius $\left(R_{e}+h\right)$ around the Earth in a plane perpendicular to the Equator plane (Figure 1.9). If we neglect air forces, on the body act the weight $W$ of the body in a direction approximately toward the center of the Earth and in the opposite direction an apparent force associated with the circular motion. The latter force is the familiar centrifugal force, $C$, which is given by (cf. Equation (1.4)

$$
\begin{equation*}
C=-M \dot{\theta} \times\left[\dot{\theta} \times\left(R_{e}+h\right)\right]=\frac{W}{g} \frac{V^{2}}{\left(R_{e}+h\right)}, \tag{1.16}
\end{equation*}
$$

where $\theta$ is latitude and $V$ is the velocity of the body.
The relative importance of the centrifugal force can be expressed by the ratio:

$$
\begin{equation*}
\frac{C}{W}=\frac{V^{2}}{\left(R_{e}+h\right) g} \tag{1.17}
\end{equation*}
$$



Figure 1.9 Flight around the Earth

The speed at which the centrifugal force equals the weight of the body is called the circular velocity $V_{c}$ :

$$
\begin{equation*}
V_{c}=\sqrt{\left(R_{e}+h\right) g} . \tag{1.18}
\end{equation*}
$$

At sea level $(h=0)$ we find from Equation (1.18), using $R_{e}=6371 \times 10^{3} \mathrm{~m}$ and $g=g_{0}=9.80665 \mathrm{~m} / \mathrm{s}^{2}: V_{c_{0}}=7904 \mathrm{~m} / \mathrm{s}=28455 \mathrm{~km} / \mathrm{h}$.
Since Figure 1.7 indicates that our analyses mostly will concern airspeeds which are small with respect to the circular velocity, it follows from Equation (1.17) that we usually can ignore the effect of the centrifugal force $(C \ll W)$ on the motion of the airplane so that the Earth can be regarded as ideally flat.

### 1.6 Coordinate systems

To describe the motion of an airplane four coordinate systems, employing righthanded, rectangular Cartesian axis systems, are used. The origin is denoted by " 0 " and the axes designated $X, Y, Z$. Displacements are positive in the positive senses of the axes and angles are positive in clockwise direction when looking along the appropriate axis in the positive direction. Velocities, angular velocities and accelerations also are positive in these directions.
a. The Earth axis system or ground axis system (Figure 1.10). Earth axes are denoted by the subscript " g ". The origin of this coordinate system is any point on the Earth's surface. The $X_{g}$ - and $Y_{g}$-axes lie in the horizontal plane of the Earth. The $X_{g}$-axis points into an arbitrary direction. E.g., the $X_{g}$-axis is taken in the direction of flight. The $Z_{g}$-axis points vertically and positive downward.
b. The moving Earth axis system or local horizon system (Figure 1.11). The axes are denoted by the subscript "e". The origin of the system is taken to be the center of gravity of the airplane. The $X_{e}, Y_{e}$ and $Z_{e}$ axes are parallel to the corresponding axes of the Earth axis system. Thus, the plane formed by the $X_{e^{-}}$and $Y_{e^{-}}$-axes is always parallel to the surface of the Earth.


Figure 1.10 Earth axis system


Figure 1.11 Moving Earth axis system
c. The body axis system or airplane axis system (Figure 1.12). Body axes are denoted by the subscript " $b$ ". The origin of the system is at the center of gravity of the airplane. The $X_{b}$-axis lies in the plane of symmetry of the airplane and points out of the nose of the airplane. The $Z_{b}$-axis is perpendicular to the $X_{b}$-axis, lies also in the plane of symmetry, and is directed downward for a normal flight attitude. The $Y_{b}$-axis is directed out of the right wing of the airplane. The body axes are fixed to the airplane and oriented by reference to some geometrical datum. The $X_{b}$-axis coincident with what is called the longitudinal axis of the airplane. The $Y_{b}$-axis usually is termed transverse or lateral axis and the $Z_{b}$-axis is named normal axis. The rotational components about $X_{b}, Y_{b}$ and $Z_{b}$ are called roll, pitch and yaw, respectively.
d. The air-path axis system or flight-path axis system (Figure 1.13). Air-path axes are denoted by the subscript "a". The origin is at the center of gravity of the airplane. The $X_{a}$-axis lies along the velocity vector. The $Z_{a}$-axis is taken in the plane of symmetry of the airplane, and is positive downward for a normal airplane attitude. Consequently, the $Y_{a}$-axis is positive to starboard.


Figure 1.12 Body axis system


Figure 1.13 Air-path axis system

### 1.7 Angles and velocities describing the angular displacement of the airplane

In order to describe the attitude of the airplane with respect to the moving Earth axis system, a number of characteristic angles are used. These angles often are


Figure 1.14 Eulerian angles
called Eulerian angles and are presented in Figure 1.14.
a. Eulerian angles defining the orientation of the airplane body axes. These angles are:

- Angle of yaw $\psi$; the angle between the projection of the $X_{b}$-axis on the $X_{e} Y_{e}$-plane (horizontal plane) and the $X_{e}$-axis.
- Angle of pitch $\theta$; the angle between the $X_{b}$-axis and its projection on the $X_{e} Y_{e}$-plane.
- Angle of roll $\phi$; the angle between the $Y_{b}$-axis and the intersecting line of the $Y_{b} Z_{b}$-plane with the $X_{e} Y_{e}$-plane.

The Eulerian angles $\psi, \theta$ and $\phi$ are obtained by three defined successive rotations of the moving Earth axes. This procedure is illustrated in Figure 1.15. First, we rotate by $\psi$ about $Z_{e}$, then by $\theta$ about $Y^{\prime}$, and finally by $\phi$ about $X_{b}$. As shown in Figure 1.14, also may be used the angle of bank $\Phi$, being the angle between the $Y_{b}$-axis and its projection on the $X_{e} Y_{e}$-plane. The angle of bank can be written in terms of the angle of roll and the angle of pitch:

$$
\sin \Phi=\sin \phi \sin (90-\theta)
$$

or

$$
\begin{equation*}
\sin \Phi=\sin \phi \cos \theta \tag{1.19}
\end{equation*}
$$

The relation (1.19) follows by applying a theorem from spherical trigonometry in the spherical triangle ABC in Figure 1.16.


- A rotation by $\theta$ about the $Y^{\prime}$-axis to the intermediate position $X_{b} Y^{\prime} Z^{\prime}$.



## - A rotation by $\phi$ about the $X_{b}$-axis to the final position $X_{b} Y_{b} Z_{b}$.

Figure 1.15 Orientation of body axes to moving Earth axes
b. Eulerian angles defining the orientation of the air-path axes. These angles are (Figure 1.14):

- Azimuth angle $\chi$; the angle between the projection of the $X_{a}$-axis on the $X_{e} Y_{e}$-plane and the $X_{e}$-axis.
- Flight-path angle $\gamma$; the angle between the $X_{a}$-axis and its projection on the $X_{e} Y_{e}$-plane.
- Aerodynamic angle of roll $\mu$; the angle between the $Y_{a}$-axis and the intersecting line of the $X_{a} Y_{a}$-plane with the $X_{e} Y_{e}$-plane.


Figure 1.16 Relation between angle of roll, angle of pitch and angle of bank


- A rotation by $\mu$ about the $X_{a}$-axis
to the final position $X_{a} Y_{a} Z_{a}$.

Figure 1.17 Orientation of air-path axes to moving Earth axes

The angles $\chi, \gamma$ and $\mu$ are also generated by three successive rotations of the moving Earth axes. The sequence of rotations is indicated in Figure 1.17. First, we


Figure 1.18 Orientation of air-path axes to body axes
rotate by $\chi$ about $Z_{e}$, then by $\gamma$ about $Y^{\prime}$, and finally by $\mu$ about $X_{a}$. Of importance is also the relationship between the air-path axis system and body axis system. Both coordinate systems are shown in Figure 1.18. Since the $Z_{a}$-axis lies in the $X_{b} Z_{b}$-plane (plane of symmetry of the airplane), the orientation of the air-path axis system with respect to the body axis system is completely defined by the following two angles:

- Angle of attack $\alpha$; the angle between the projection of the $X_{a}$-axis on the plane of symmetry of the airplane and the $X_{b}$-axis.
- Angle of sideslip $\beta$; the angle between the $X_{a}$-axis and its projection on the plane of symmetry of the airplane.

The angle of attack is positive when the velocity component along the $Z_{b}$-axis is positive. The angle of sideslip is positive when the velocity component along the $Y_{b}$-axis is positive.
At this point it is suited to define the components of the airspeed $V$ along the $X_{b}$, $Y_{b}$ and $Z_{b}$ axes of the body axis system as $u, v$ and $w$, respectively (Figure 1.19). The following relations are apparent:


Figure 1.19 Components of airspeed

$$
\left.\begin{array}{l}
V^{2}=u^{2}+v^{2}+w^{2}  \tag{1.20}\\
u=V \cos \beta \cos \alpha \\
v=V \sin \beta \\
w=V \cos \beta \sin \alpha
\end{array}\right\} .
$$

Similarly, the resultant angular velocity $\Phi$ can be resolved into the components $p$, $q$ and $r$ along the $X_{b}, Y_{b}$ and $Z_{b}$ axes, respectively, where

$$
\begin{equation*}
\Phi^{2}=p^{2}+q^{2}+r^{2} . \tag{1.21}
\end{equation*}
$$

The angular veiocity about the $X_{b}$-axis is the rolling velocity, positive if the right wing drops. The angular velochy about the $Y_{b}$-axis is the pitching velocity, positive if the nose of the airplane rises. The angular velocity about the $Z_{b}$-axis, finally, is the yawing velocity, positive if the nose of the airplane moves to the right (clockwise when observed from above).
In connection with the study of airplane motion, the relations between the angular velocities, $p, q, r$, about the body axes and the time rate of change of the Eulerian angles $\psi, \theta, \phi$, may be of importance.
According to the rotations defined in Figure 1.15, the vectors $\frac{d \psi}{d t} . \frac{d \theta}{d t} . \frac{d \phi}{d t}$ are directed along the $Z_{e}, Y^{\prime}$ and $X_{b}$ axes, respectively.
Figure 1.20 shows these vectors. Resolving along the body axes leads to the following relationships between the two sets of angular velocities:

$$
\left.\begin{array}{l}
p=-\frac{d \psi}{d t} \sin \theta+\frac{d \phi}{d t}  \tag{1.22}\\
q=\frac{d \psi}{d t} \cos \theta \sin \phi+\frac{d \theta}{d t} \cos \phi \\
r=\frac{d \psi}{d t} \cos \theta \cos \phi-\frac{d \theta}{d t} \sin \phi
\end{array}\right\}
$$

and the inverse relationships:

$$
\left.\begin{array}{l}
\frac{d \psi}{d t}=\frac{1}{\cos \theta}(q \sin \phi+r \cos \phi)  \tag{1.23}\\
\frac{d \theta}{d t}=q \cos \phi-r \sin \phi \\
\frac{d \phi}{d t}=p+\frac{d \psi}{d t} \sin \theta=p+(q \sin \phi+r \cos \phi) \tan \theta
\end{array}\right\}
$$

In the presence of wind the velocity of the airplane with respect to the ground or ground speed $V_{g}$ is the vector sum of the speed of the airplane relative to the air $V$ and the wind velocity $V_{w}$ (Figure 1.21):

$$
\begin{equation*}
\vec{V}_{g}=\vec{V}+\vec{V}_{w} . \tag{1.24}
\end{equation*}
$$

Since the airplane is carried along by the wind, the projection of the velocity vector $V$ on the ground is at a so-called drift angle with the actual flight track.
Figure 1.22 shows the components of the airspeed along the axes of the moving Earth axis system. From this figure we obtain:

$$
\left.\begin{array}{l}
V_{X_{e}}=V \cos \gamma \cos \chi  \tag{1.25}\\
V_{Y_{e}}=V \cos \gamma \sin \chi \\
V_{Z_{e}}=V \sin \gamma
\end{array}\right\} .
$$



Figure 1.20 Angular velocities about the body axes
Note that a positive sign is given to the component of $V$ in the direction of the negative $Z_{e}$-axis. Hence we find the components of the ground speed along the axes of the Earth axis system as:

$$
\left.\begin{array}{l}
V_{X_{g}}=V \cos \gamma \cos \chi+u_{w}  \tag{1.26}\\
V_{Y_{g}}=V \cos \gamma \sin \chi+v_{w} \\
V_{Z_{g}}=V \sin \gamma+w_{w}
\end{array}\right\}
$$



Figure 1.21 Ground speed


Figure 1.22 Components of airspeed

In Equation (1.26), wind data are given as $u_{w}, v_{w}, w_{w}$, being the respective components of the wind velocity in terms of the moving Earth axis system. The positive sense of $w_{w}$ is taken in upward direction.

### 1.8 The airplane

Figure 1.23 shows in some detail the overall make-up of an airplane. Basic components are fuselage, wing, tail assembly, controls, landing gear, and engine (and propeller, in the case of propeller propulsion).


Figure 1.23 Basic airplane components


Figure 1.24 Balance and trim tab

The fuselage may be seen as the structural component to which the other main parts are connected. Further, it provides space for crew, passengers, cargo, airplane systems and instrumentation. Generally, the fuselage is streamlined to reduce its drag.
The wing is the principal component to generate the lift of the airplane by its motion with respect to the surrounding air. The wing may often be equipped with flaps. These adjustable parts are used to increase lift and drag at low airspeeds.
The tail assembly consists of the vertical and the horizontal stabilizer, which surfaces provide directional stability in yaw and stability in pitch, respectively.
Included in the tail assembly and the wing are the control surfaces. The usual position of the three primary controls is also illustrated in Figure 1.23. Yaw control is provided by the rudder, which is connected with the vertical stabilizer.
The elevators are attached to the horizontal stabilizer and control the pitch of the airplane. Roll control is provided by deflections of the ailerons which are located near the outer trailing edges of the wing.
Depending on the type of airplane, small auxiliary control surfaces may be in-


Figure 1.25 Landing gear types
stalled to the trailing edges of the elevators, rudder and ailerons. These movable surfaces are known as trim tabs and are adjusted by the pilot. As shown in Figure 1.24, the airflow over the trim tab creates a moment that holds the primary control surface in the desired position without any help from the pilot.
Tabs may also be used to assist the pilot in the movement of the primary controls; these are known as balance tabs.
The landing gear or undercarriage supports the airplane while it is in contact with the ground. Modern airplanes generally are equipped with a tricycle gear, consisting of nose wheel and main wheels. The landing gear may be retractable, except special forms which include skis for snow and floats for operations on water (Figure 1.25).
An important characteristic is the type of propulsion system. The main engine types are the piston engine (reciprocating engine), and the reaction engine such as turbojet, turboprop, and turbofan. Converting the power of a piston engine and a turboprop into a thrust is accomplished by the propeller(s).

### 1.9 Flight types, airplane configuration and flight condition

In Figure 1.26 are illustrated the typical flight phases encountered by an airplane during a trip over a given travel distance.
The takeoff consists of the takeoff run where the airplane is accelerated from standstill to the liftoff speed, followed by the climbout to a distance over, say, 10.7 m ( 35 ft ) obstacle. After the takeoff the power of the engine(s) is reduced and the airplane climbs to cruise altitude at, approximately, constant velocity. The latter conditions hold as much with regard to the descent. Also cruise flight is executed in unaccelerated and straight flight.
An example of a curved flight path is the turn and particularly the so-called constant-altitude banked turn, where the airplane is inclined about the longitudinal axis. This type of turn is the usual manner in which the flight path heading is changed, e.g., in the holding maneuver. As depicted in Figure 1.26, during holding the airplane remains within a specified airspace whilst awaiting further clearance for the approach flight to the airport runway.
The final flight phase, naturally, is the landing, proceeding from the steady approach flight so as to clear the screen height at the beginning of the runway and to come to rest on the runway at the end of the ground run. Just like the takeoff, the landing is a case of unsteady airplane motion.


Figure 1.26 Typical flight phases

In the various flight phases, usually, the airplane is controlled in such a manner that the instantaneous motion satisfies certain conditions. This leads to well-defined flight types, such as:

- Gliding flight; flight in which the thrust is zero.
- Steady flight; flight in which the forces and moments acting on the airplane do not vary in time, neither in magnitude nor in direction.
- Nonsideslipping flight; flight in which the velocity vector is parallel to the plane of symmetry of the airplane (angle of sideslip is zero).
- Straight flight; flight in which the center of gravity of the airplane travels along a straight line.
- Symmetric flight; flight in which both the angle of sideslip is zero and the plane of symmetry of the airplane is perpendicular to the horizontal plane of the Earth.

At this point, it is useful to emphasize that symmetric flight, and in particular steady symmetric flight forms the basis of considerations on the performance of airplanes during most of their time of flying. In this connection, it may be clear that at best an airplane can perform a quasi-steady flight due to the consumption of engine fuel and/or the variation of atmospheric conditions.
The term airplane configuration or airplane condition indicates the description of the external shape of the airplane and any parameter affecting the motion of the airplane which is characterized by the fact that it remains constant during a certain period of time. Examples of airplane configuration elements are (Figure 1.27) landing gear position, flap angle, speedbrake and spoiler deflections, and number of operative engines.
The estimation of airplane performance may be treated by considering the airplane in a given configuration which is related to a particular flight phase, such as takeoff configuration, cruise configuration and landing configuration.
The term flight condition is the group of variables, which defines the motion of the airplane at each instant of the flight. A description of the flight condition will comprise airplane weight, altitude, atmospheric conditions, airspeed, power setting and control surface deflections.

### 1.10 Forces on the airplane

Practically, there are two different kinds of external forces that act on an airplane in flight, gravity forces and aerodynamic forces.


Figure 1.27 Example of airplane configuration

Gravity forces are related to the mass of a body and act from a distance. A common example is, of course, the weight of the airplane.
Aerodynamic forces are developed through application of Newton's third law of motion, which states that for every action there is an equal and opposite reaction (Appendix A). Therefore, essential to the generation of an aerodynamic force is the occurrence of relative motion between body and medium.
In this course book we shall use the symbol $R$ to denote the aerodynamic force produced by the interaction between the air and the outer surface of the airplane. When resolved into components along the air-path axes, the vector force $R$ delivers the lift, drag, and side force. The lift, designated by the symbol $L$, is the component along the negative $Z_{a}$-axis. The major portion of the lift arises from the airflow around the wing. The drag $D$ and side force $S$ are the components of the aerodynamic force $R$ along the negative $X_{a}$-axis and $Y_{a}$-axis, respectively. A side force or cross force appears only when the airplane is in sideslipping flight. Figure 1.28 shows the aerodynamic force $R$ in the case of symmetric flight. In this type of flight the motion is in the geometric plane of symmetry so that besides the $X_{b}$-axis, also the $X_{a}$-axis lies in the plane of symmetry of the airplane.
When studying rotational motion in symmetric flight it may be useful to employ the tangential force $T$ and the normal force $N$ being the components of $R$ along the negative $X_{b}$-axis and $Z_{b}$-axis, respectively. As can be seen from Figure 1.28 the components $L$ and $D$, and the components $N$ and $T$ are related by the expressions:

$$
\left.\begin{array}{l}
L=N \cos \alpha-T \sin \alpha  \tag{1.27}\\
D=N \sin \alpha+T \cos \alpha \\
N=L \cos \alpha+D \sin \alpha \\
T=-L \sin \alpha+D \cos \alpha
\end{array}\right\}
$$

where $\alpha$ is the angle of attack.
Also the driving force of the propulsion system is an aerodynamic force. This force is called the thrust and also given the symbol $T$.
As indicated in Figure 1.29, the thrust acts in forward direction along a working line which makes a fixed angle $\eta$ with the longitudinal axis of the airplane ( $X_{b}$ axis). The type of flight considered in Figure 1.29 represents the case of steady symmetric flight. Maintaining this type of flight requires that the vector sum of the forces acting on the airplane is zero:

$$
\begin{equation*}
\vec{R}+\vec{T}+\vec{W}=0 \tag{1.28}
\end{equation*}
$$



Figure 1.28 Aerodynamic force $R$ and components


Figure 1.29 Forces In steady symmetric flight

The general definitions of the various angles used in Figure 1.29, i.e., the angle of attack a, the flightpath angle $\gamma$ and the angle of pitch $\theta$, have been given in Section 1.7.

### 1.11 SI-system of units

Throughout this book the International System of Units (Système International d'Unités) is used. This system has been adopted by many countries as the recommended system of units for weights and measures. According to the publications of the International Organization for Standardization (ISO) there are seven basic units, which are tabulated in Table 1.1.
Although the SI-unit of temperature is the kelvin (K), also the celsius (C) or centigrade scale is used. Since the unit degree celcius is exactly equal to the unit kelvin, the temperature expressed in degree celsius can be readily converted to the absolute temperature in kelvin by the following relationship,

$$
\begin{equation*}
\text { degree } C=K-273.15 \tag{1.29}
\end{equation*}
$$

From the basic units in Table 1.1, the units of a wide range of quantities can be derived, whereby the product and/or quotient of any number of basic units forms the resultant unit of the derived quantity. The units of some of the more common quantities are listed in Table 1.2.
To obtain multiples or decimal fractions of the units, standard prefixes are used, which are collected in Table 1.3.
In order to prevent errors in calculations, it is strongly recommended that in computations only SI-units are used and not their multiples or decimal fractions.
lt should be mentioned that, though becoming obsolete, in engineering practice frequently the so-called technical system of units is used. In this system the quantity force, having also the name kilogram, is a basic unit instead of mass. In order to distinguish both kilograms, in the technical system the quantity force is (often) denoted as kilogramforce (abbreviation: kgf). The following relation is defined: $1 \mathrm{kgf}=9.80665 \mathrm{~N}$. Some technical units and corresponding SI-units are given in Table 1.4.
In Appendix B a number of conversion factors are collected, arranged according to subject categories.

Table 1.1 Basic SI-units

| quantity | name of unit | symbol |
| :--- | :--- | :--- |
| length | meter | m |
| mass | kilogram | kg |
| time | second | s |
| temperature | kelvin | K |
| electric current | ampère | A |
| luminous intensity | candela | cd |
| amount of substance | mole | mol |

Table 1.2 Derived SI-units

| quantity | name of unit | symbol | definition |
| :--- | :--- | :--- | :--- |
| force | newton | N | $\mathrm{kg} \mathrm{m} / \mathrm{s}^{2}$ |
| pressure | pascal | Pa | $\mathrm{N} / \mathrm{m}^{2}$ |
| work (energy) | joule | $J$ | $\mathrm{~J}=\mathrm{N} \mathrm{m}$ |
| power | watt | $W$ | $\mathrm{~J} / \mathrm{s}$ |
| velocity | meter per second | $V$ | $\mathrm{~m} / \mathrm{s}$ |
| acceleration | meter per second squared | $a$ | $\mathrm{~m} / \mathrm{s}^{2}$ |
| moment of force | newton meter | $M$ | N m |
| density | kilogram per unit cubic meter | $\rho$ | $\mathrm{kg} / \mathrm{m}^{3}$ |

Table 1.3 Standard muitiples and decimal fractions

| multiple/fraction | prefix | symbol |
| :--- | :--- | :--- |
| $10^{12}$ | tera | T |
| $10^{9}$ | giga | G |
| $10^{6}$ | mega | m |
| $10^{3}$ | kilo | k |
| $10^{2}$ | hecto | h |
| 10 | deca | da |
| $10^{-1}$ | deci | d |
| $10^{-2}$ | centi | c |
| $10^{-3}$ | milli | m |
| $10^{-6}$ | micro | $\mu$ |
| $10^{-9}$ | nano | n |
| $10^{-12}$ | pico | p |
| $10^{-15}$ | femto | f |
| $10^{-18}$ | atto | a |

Table 1.4 Systems of units

| quantity | technical system metric | English | SI-system |
| :---: | :---: | :---: | :---: |
| length | m | ft | m |
| time | s | S | s |
| force | kgf | lbf | $\mathrm{kg} \mathrm{m} / \mathrm{s}^{2}$ (newton) |
| mass | kgf s ${ }^{2} / \mathrm{m}$ | $\mathrm{lbf} \mathrm{s}^{2} / \mathrm{ft}$ (slug) | kg |
| pressure | $\mathrm{kgf} / \mathrm{m}^{2}$ | $\mathrm{lbf} / \mathrm{ft}^{2}$ | $\mathrm{N} / \mathrm{m}^{2}$ |
| work (energy) | kgf m | 1 bfft | $\mathrm{kg} \mathrm{m}{ }^{2} / \mathrm{s}^{2}=\mathrm{Nm}$ (joule) |
| power | kgf m/s | $\mathrm{lbfft} / \mathrm{s}$ | $\mathrm{kg} \mathrm{m} / \mathrm{s}^{3}=\mathrm{J} / \mathrm{s}$ (watt) |
| density | $\mathrm{kgf} \mathrm{s}{ }^{2} / \mathrm{m}^{4}$ | $\mathrm{lbf} \mathrm{s}{ }^{2} / \mathrm{ft}^{4}$ | $\mathrm{kg} / \mathrm{m}^{3}$ |

