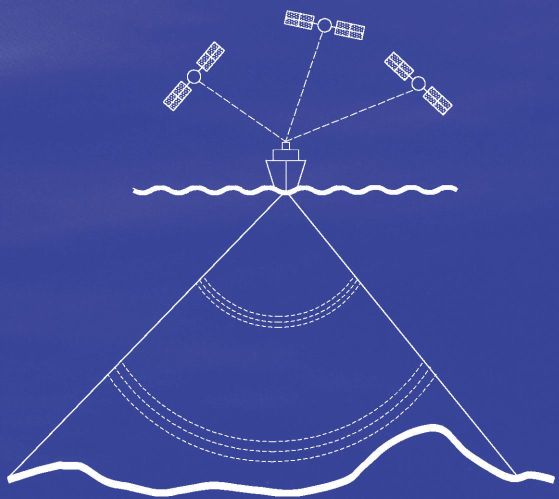
Hydrography

C.D. de Jong, G. Lachapelle,
S. Skone, I.A. Elema



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Hydrography

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C.D. de Jong
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Preface

This book is based on the lecture notes for the graduate and undergraduate courses in hydrography as offered at the Department of Geomatics Engineering of the Universitiy of Calgary and the Department of Mathematical Geodesy and Positioning of Delft University of Technology. The purpose of the book is to present an introduction to and an overview of the broad field of hydrography. Since there is only a weak interdependence between the eleven chapters, each of them can be studied separately. When used for a course, it is therefore also possible to consider only a selected number of chapters.

Dr. Jan Krynski, Mr. Mark Petovello and Mr. Kyle O'Keefe are acknowledged for their assistance in adding explanatory text and correcting specific segments of the original lecture notes. The assistance of Ms. Ria Scholtes and Mr. Jacques Schievink in preparing the final version of the book is also gratefully acknowledged.

October 2002,

C.D. de Jong
Delft University of Technology
Delft, The Netherlands

G. Lachapelle
University of Calgary
Calgary, Canada

S. Skone
University of Calgary
Calgary, Canada

I.A. Elema
Netherlands Hydrographic Service
The Hague, The Netherlands

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1 Elements of oceanography

1.1 Water

About 96.5% of the contents of the oceans is water, H_2O. The remaining 3.5% consists of dissolved salts. The two H^+ atoms of a water molecule are connected to the O^{2-} atom in such a way that the two valences of oxygen join the valences of the two hydrogen atoms at an angle of 105°, see Figure 1.1. A consequence of this asymmetric distribution of electric charges is a strong dipole moment. This strong dipole moment, in turn, results in a number of special properties of water:

- Pure water has the highest dielectric constant ε of all liquids. This is of importance for the behaviour of dissolved substances: the larger ε, the smaller the attractive forces between positively charged cations and negatively charged anions.
- There is a great associative power of water molecules, which leads to the formation of molecular groups. This process is called polymerisation. For example, at 0°C, the average group size is six H_2O molecules. These polymers determine to a great extent the physical properties of water, such as the strong surface tension and viscosity and high melting and boiling temperatures.

Water also has a very large heat capacity. This large capacity enables the oceans to

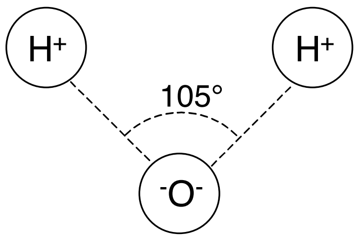

Figure 1.1: H_2O molecule.

Table 1.1: Composition of sea water with a salinity of 35 ppt.

Cations	**gr/kg**	**Anions**	**gr/kg**
Sodium	10.752	Chlorine	19.345
Potassium	0.39	Bromine	0.066
Magnesium	1.295	Fluorine	0.0013
Calcium	0.416	Sulphate	2.701
Strontium	0.013	Bicarbonate	0.145
		Boric acid	0.027

store great quantities of heat, which can be released at differrent places and times. Finally, one of the best known properties of water is the increase in volume while freezing. Pure water has its highest density at 4°C. At 0°C, the volume increases by about 9%. As a results, ice will float on water.

In the open ocean salinity varies between 3.4-3.8%, or, as it is more commonly expressed, 34-38 ppt (parts per thousand). For land-locked adjacent seas, the salinity may differ considerably. In humid zones with strong river run-offs from land it is much lower. For example, salinity in the Baltic Sea can be as low as 0.5-1 ppt. In arid regions, where evaporation exceeds precipitation, salinity is much higher, e.g., 43-45 ppt in the Red Sea. Despite the large variations in salinity, the relative proportions of its constituents are constant to within a few percent. The major constituents of sea water with a salinity of 35 ppt are shown in Table 1.1.

The density ρ of sea water depends on salinity *S*, temperature *T* and pressure *p* (relative to atmospheric pressure). Small differences in density may already result in significant sea level differences and currents. In practice, i.e., on board ships, density cannot be measured directly from the mass of a volume of water. Instead, it is derived from measurements of salinity, temperature and pressure. Since density does not change very much, the quantity σ is introduced, defined as

$$\sigma = (\rho - 1) \cdot 10^3 \tag{1.1}$$

At sea level (*p*=0) and for a temperature of 0°C, σ_0 is given by the empirical relationship

$$\sigma_0 = -0.0093 + 0.8149 \cdot S - 0.000482 \cdot S^2 + 0.0000068 \cdot S^3 \tag{1.2}$$

where *S* is expressed in ppt and σ_0 in 10^3 kg/m^3.

Pure water has its maximum density at 4°C. The temperature of maximum density decreases with increasing salinity according to

$$T_{\rho_{\max}} = 3.95 - 0.266 \cdot \sigma_0 \tag{1.3}$$

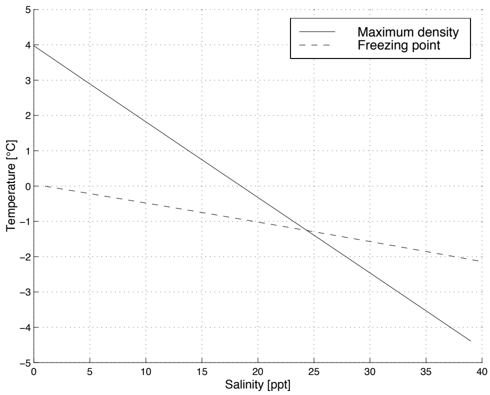

Figure 1.2: Temperature of maximum density and freezing point for sea water of different salinities.

where the temperature is expressed in °C. Thus, for salinities above 18 ppt, the temperature of maximum density is below the freezing temperature of pure water. Due to its salt content, the freezing temperature of sea water is below that of pure water. This temperature T_g, in °C, as a function of salinity is given by the relationship

$$T_g = -0.0086 - 0.064633 \cdot \sigma_0 - 0.0001055 \cdot \sigma_0^2 \tag{1.4}$$

As can be seen from Figure 1.2, the temperature of maximum density and the freezing temperature are the same (-1.33°C) for a salinity of 24.7 ppt. If the salinity is less than 24.7 ppt, the temperature of maximum density, with cooling of the water, is reached before the freezing temperature. At a given temperature above the freezing point, the water from the surface to the bottom has reached its maximum density. A little more cooling of the surface layers results in water at the surface that is lighter than the subsurface waters and therefore does not sink. Eventually, when the freezing point is reached, an ice sheet is formed. If the salinity is above 24.7 ppt, vertical convection continues with coooling until the entire water column has reached its freezing point. Thus, the cooling of water of a high salinity extends to a much greater depth and to much lower temperatures than in the case of low salinities.
In conclusion, it is not just the depression of the freezing point of the sea water that explains why the salty sea does not freeze as rapidly as, e.g., fresh water lakes or seas of low salinity, but rather the relationship between the temperature of density maximum and freezing temperature.

1.2 Ocean currents and general circulation

When the atmosphere and the ocean meet, the energy from the moving air is passed to the water through friction. It results in movement of the surface layer of water due to the drag exerted by winds blowing steadily across the ocean. The major horizontal

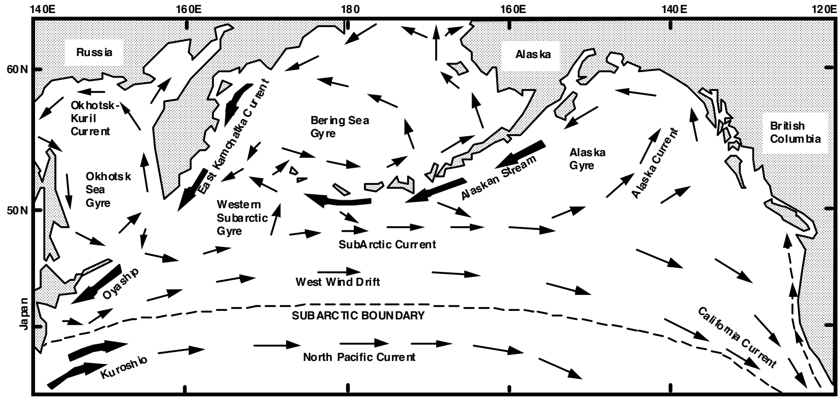

Figure 1.3: Surface currents in the North Pacific Ocean.

movements of the surface waters, the *surface circulations*, are closely related to the general circulation of the atmosphere, which is generated by the unequal heating of the Earth by the Sun. *Surface currents* are wind driven. They occur in the most upper layer of the ocean, i.e., the first 1000 m of water. The Gulf Stream in the Atlantic Ocean is an example of a surface current. *Thermohaline circulation* refers to the water movement which takes place when the density is changed by a change of temperature (thermocline) or of salinity (halocline). Heat combined with salinity causes vertical density variations which results in vertical circulation sometimes called *thermohaline currents*. Ocean currents are a result of surface circulations and thermohaline circulations. Surface currents in the North Pacific Ocean are shown in Figure 1.3. The winter and summer surface circulation in the Pacific Ocean off the coasts of British Columbia and Washington is shown in Figure 1.4 while the general ocean circulation is shown in Figure 1.5.

A number of major current types can be distinguished. *Sub-tropical gyres* span the entire east-west dimension of each ocean basin in both the northern and southern hemispheres (e.g. Gulf Stream). Due to the Coriolis effect as well as surface wind stress, the gyres are:

- Clockwise in the northern hemisphere, e.g., Gulf Stream/Canary/North Equatorial

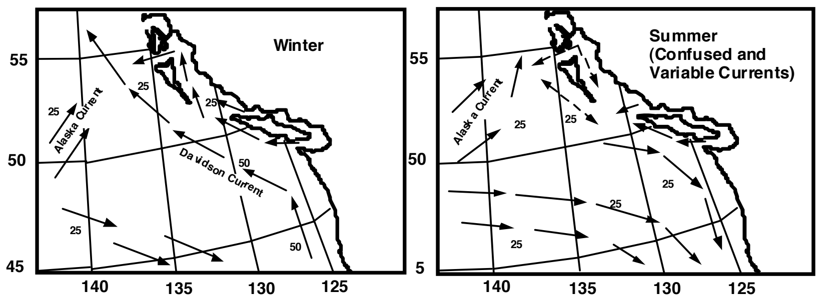

Figure 1.4: Winter (left) and summer circulation off British Columbia and Washington State coasts.

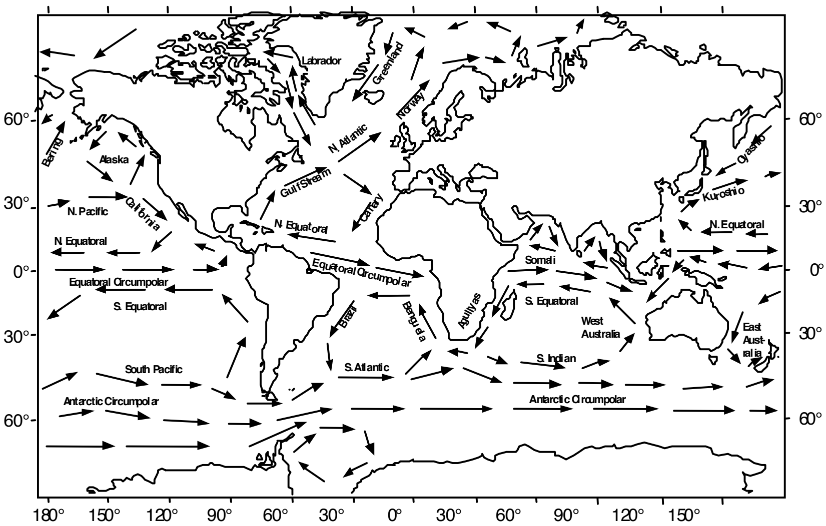

Figure 1.5: General ocean circulation.

Current.

- Counterclockwise in the southern hemisphere, e.g. South Equatorial/Brazil/South Atlantic/Benguela.

Equatorial currents cross the ocean in easterly direction along the equator. The *Antarctic Circumpolar current* flows eastward. The velocity and volume of major currents are given in Table 1.2.

The main factor generating the surface currents is the wind, which is related to the general circulation of the atmosphere. Atmospheric circulation in turn is generated by the unequal heating of the Earth by the Sun. The energy which is gained by the Earth is equal to that lost to space. There is a net gain of energy in the low latitudes and a net loss at higher latitudes. Since neither polar regions nor the tropics progressively change temperature, there must be a transfer of heat from areas of excess to areas of deficit. This transfer is done by means of winds (about 75% of heat transfer) and ocean currents (about 25% of heat transfer). Solar radiation accounts for more than

Table 1.2: Velocity and transport of major ocean currents.

Current	**Maximum velocity (cm/s)**	**Transport (10^6 m^3/s)**
Gulf Stream	200-300	100
North Equatorial Pacific	20	45
Kuroshio	200	50
Equatorial Undercurrent	100-150	40
Brazil	-	10
Antarctic Circumplolar	-	100
Peru (or Humboldt)	-	20

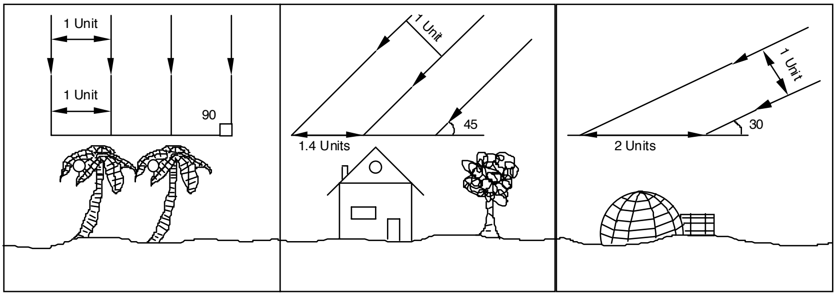

Figure 1.6: Variations in solar energy reaching the Earth's surface.

99% of the energy that heats the Earth.

The seasonal variations in the elevation of the Sun relative to the equator affect the amount of energy received at the Earth's surface in two ways:

- The lower the elevation angle, the more spread out and less intense is the solar radiation that reaches the surface (most important factor).
- The lower the elevation angle, the longer the path of the rays through the atmosphere, and the greater absorption, reflection and scattering by the atmosphere, which reduces the intensity at the surface (less important factor).

Changes in the Sun elevation angle cause variations in the amount of solar energy reaching the Earth's surface, as shown in Figure 1.6. At higher latitudes, the Sun rays strike the Earth at a lower angle and must traverse more of the atmosphere: this results in greater depletion by reflection and absorption, as shown in Figure 1.7.

Two models of global circulation will be considered: a model for a non-rotating Earth and a model for a rotating Earth. In the *non-rotating Earth model*, more intensely heated equatorial air rises and moves poleward. This upper level flow reaches the poles, where it sinks and spreads out at the surface and returns to the equator. Cold polar air moves towards the equator on the Earth's surface, becomes re-heated, and rises again, as shown in Figure 1.8. In the *rotating Earth model*, the effect of Coriolis forces on winds results in the general patterns shown in Figure 1.9.

The effects of Coriolis forces on currents follow from the equation of motion in oceanography, derived from Newton's second law, which states that the observed

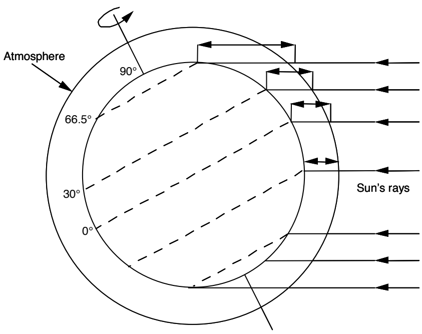

Figure 1.7: Angles of incidence of Sun rays.

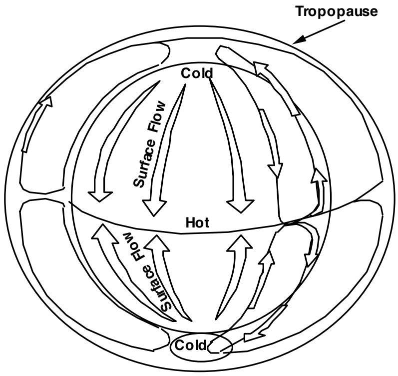

Figure 1.8: Pole-equator air movement.

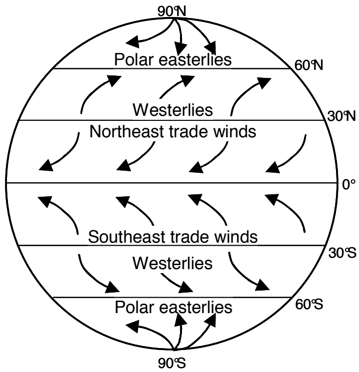

Figure 1.9: Effect of Coriolis force on winds.

acceleration is due to the resultant force acting per unit mass. This acceleration is the sum of the forces due to pressure, Coriolis effect, gravity, friction and tides, divided by a unit mass.

The *Coriolis force* f_C is defined as

$$f_C = -2\omega \times v \tag{1.5}$$

where ω is a vector of angular velocity of the Earth rotation about its spin axis, parallel to the spin axis, and pointing towards the north pole and v is the vector of velocity of the unit mass, with respect to the Earth. The Coriolis force is thus present only if the particle is *in motion* on the *rotating* Earth. The surface velocity of the current causes an additional force, i.e., a Coriolis force, that is normal to both vectors ω and v. The horizontal component of the Coriolis force (the one which causes the water flow) varies from zero at the equator to a maximum at the poles.

If the velocity vector v points eastwards, then the Coriolis force f_C points outside the Earth; if it points westwards, the Coriolis force points inside the Earth. The horizontal component of the Coriolis force will therefore always be to the right of v in the northern hemisphere, and to the left of v in the southern hemisphere. Thus, in view of the effect of the predominant winds (which themselves are also affected by the Coriolis force), the subtropical gyres are, see also Figure 1.5, clockwise in the northern hemisphere and counterclockwise in the southern hemisphere.

The horizontal component $f_{C,h}$ of the Coriolis force per unit mass (or horizontal component of the Coriolis acceleration) is

$$f_{C,h} = 2 \cdot \|\omega\| \cdot v_h \cdot \sin\varphi \tag{1.6}$$

where $\|\omega\| = 7.29 \cdot 10^{-5}$ rad/s, v_h is the horizontal component of the velocity vector v of the unit mass with respect to the Earth and φ is latitude.

Example 1.1

Consider a current speed of 1 m/s (2 knots – typical major current). Then

$f_{C,h} = 1.5 \cdot 10^{-4}$ m/s^2 at the pole ($\varphi = 90°$)

$f_{C,h} = 1.0 \cdot 10^{-4}$ m/s^2 at latitude $\varphi = 45°$

$f_{C,h} = 0$ m/s^2 at the equator ($\varphi = 0°$)

The concepts of geopotential surface and isobaric surface are now introduced in order to discuss the geostrophic method of calculating currents. A *geopotential surface* is the surface on which the gravity potential is constant (gravity potential is the same at all points on the geopotential surface). A geopotential surface is also called a *level surface*. An example of a geopotential surface is the smooth surface of a calm lake (with no currents and no waves).

An *isobaric surface* is a surface on which the pressure is constant (pressure is the same at all points on the isobaric surface). An example of an isobaric surface is the smooth surface of a calm lake, with no currents and no waves, and with the atmospheric pressure being constant. Such a surface is both a geopotential and an isobaric surface. Isobaric surfaces are level in the stationary state.

The currents resulting from the Coriolis forces are called *geostrophic* (Earth-turning) flows. In the absence of friction they move approximately along the surfaces of constant elevation (constant dynamic height). Consider the non-stationary case illustrated in Figure 1.10, where the isobaric surfaces do not coincide with level surfaces. An isobaric surface (dashed line) is inclined to the level surface (full line) by an angle *i*. A and B are particles of water of unit mass.

The pressure force on the particle A is $\alpha \cdot \partial p / \partial n$, where α is the reciprocal of the density and $\partial p / \partial n$ is the pressure gradient along the normal to the isobaric surface (isobar); it should be viewed in the plane of the paper. Besides the pressure, gravity also acts on the particle A. At A, the total pressure and gravity forces are not parallel. Thus these two forces do not balance. There is a resultant force component to the left.

The horizontal and vertical components of the pressure force are shown for particle B:

- The horizontal component: $\alpha \cdot \partial p / \partial n \cdot \sin i$ which is unbalanced and causes accelerated motion to the left.
- The vertical component $\alpha \cdot \partial p / \partial n \cdot \cos i$ which balances with the gravity force g, i.e., $\alpha \cdot \partial p / \partial n \cdot \cos i = g \approx 9.81$ m/s^2.

The horizontal component to the left can be expressed as

$$\alpha \cdot \frac{\partial p}{\partial n} \cdot \sin i = \alpha \cdot \frac{\partial p}{\partial n} \cdot \cos i \cdot \frac{\sin i}{\cos i} = g \cdot \tan i$$

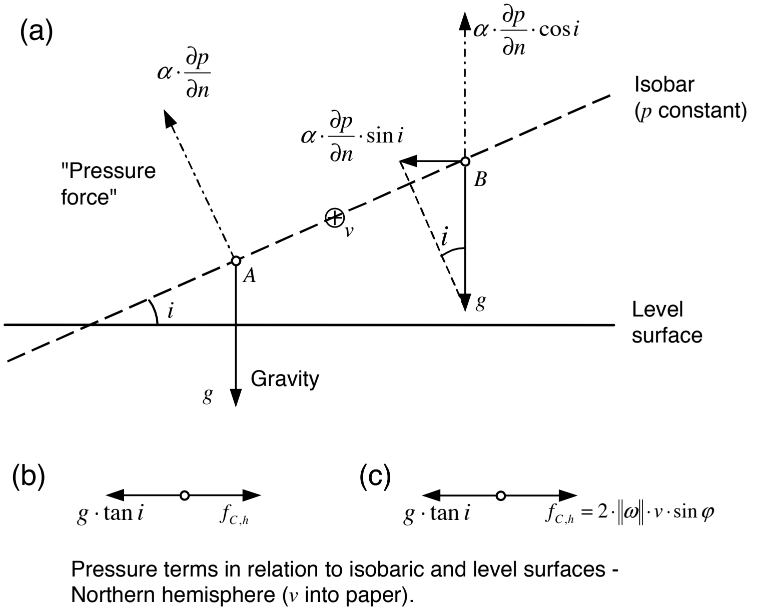

Figure 1.10: Geostrophic equation.

To balance the acceleration to the left, it is necessary to apply the same acceleration to the right. Such acceleration is generated by the Coriolis force by having the water move into the paper (in the Northern hemisphere), or out of the paper (in the Southern hemisphere) at speed v. The requirement for such a balance generates ocean current. The horizontal component to the left is thus balanced by a Coriolis force component $2 \cdot \|\omega\| \cdot v \cdot \sin i$ such that

$$2 \cdot \|\omega\| \cdot v \cdot \sin \varphi = g \cdot \tan i \qquad (1.7)$$

The above equation is called the *geostrophic equation*. It expresses a balance between pressure and Coriolis forces in a current.

Example 1.2
Consider the current with speed v=1 m/s at latitude $\varphi = 45°$. The departure (sea slope i) of the sea level from the level or geopotential surface of the current follows from (1.7) as

$$i = \tan^{-1} \frac{2 \cdot \|\omega\| \cdot \sin 45°}{g} = 10^{-5} \text{ rad}$$

i.e., 1 m in 100 km.

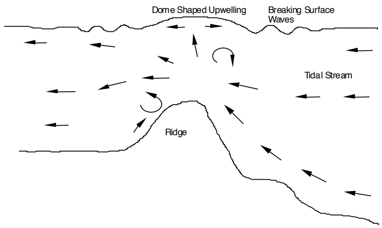

Figure 1.11: Upwelling caused by cold water.

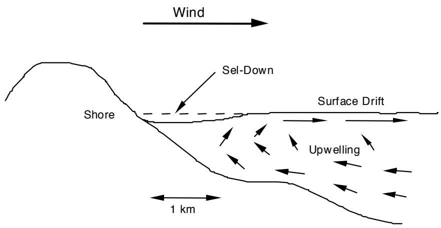

Figure 1.12: Upwelling caused by leeward winds.

Satellite Altimetry has been used successfully to estimate the sea slope of major ocean currents. This data can then be used to determine the direction and velocity of currents. In case of the Gulf Stream, the sea slope $i = (1.2 \pm 0.3) \cdot 10^{-5}$ rad. This results in a sea surface elevation of 140 cm ± 35 cm, based on SEASAT radar altimetry data collected during the late 1970's.

Upwelling is a vertical movement of cold water from deeper layers to replace warmer surface water. It affects fisheries, weather and current patterns in many parts of the world. Shown in Figure 1.11 is cold water that forces its way upward and pushes the warmer water away. On a local scale, leeward winds push the water away from the shore, as shown in Figure 1.12. On a large scale, predominant winds, combined with Coriolis forces, push warm surface water away, and cold water rises to replace it, see Figure 1.13.
Measurements, using radioactive tracer techniques have shown that the ocean's vertical circulation brings its interior water into contact with the atmosphere every 600 years. In this "overturning" circulation, cold, dense water sinks near the poles and is replaced by warmer water flowing poleward from low latitudes. In northern

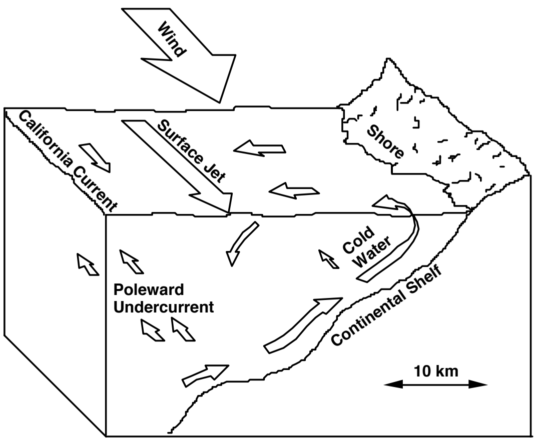

Figure 1.13: Upwelling caused by predominant winds and Coriolis forces.

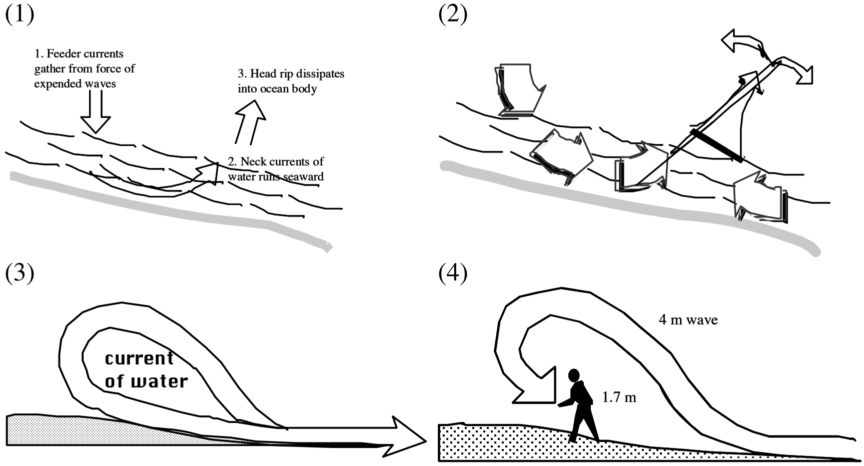

Figure 1.14: Rip currents.

and southern areas, the ocean gives up large amounts of heat to the atmosphere, namely on the order of 50 W/m^2, an amount equal to the solar energy reaching the surface in those areas. This in itself has a major impact on climate. By the time the water reaches the poles, it is cold and dense and sinks to the bottom. This overturning circulation has a major consequence on the Earth's carbon cycle and on the cycling of nutrients in the ocean. Much remains to be understood on the characteristics of these currents.

Rip currents occur when two opposing currents meet, setting up a swirling action which can carry swimmers away from the safety of shallow waters. The cycle is shown in Figure 1.14. The powerful current retreats and pulls the victim out to sea. Rip currents are the primary cause for 80% of ocean rescues. Look for (a) rough, choppy water and/or (b) deeper, darker water, (c) debris, kelp or sand.

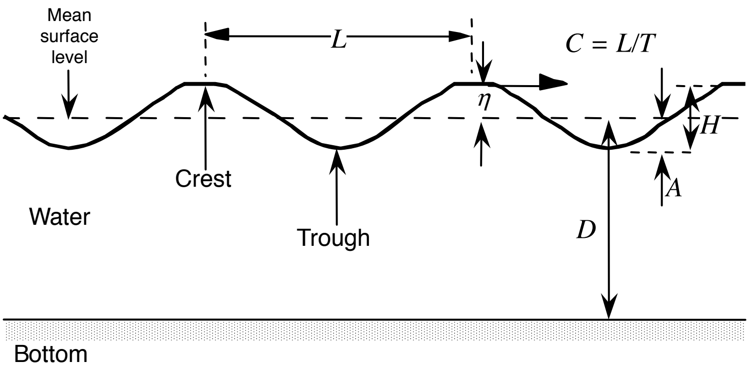

Figure 1.15: Wave characteristics.

Table 1.3: Depth range of waves.

Feature	Range	Examples
Deep water	$D > 1/4L$	Sea and swell in open oceans.
Intermediate water	$1/4L > D > 1/20L$	Long swell over shelf, sea outside surf zone.
Shallow water	$D < 1/20L$	Tsunamis, tides, swell near shore.

1.3 Waves

The characteristics of a wave depend on the following three factors:

- The type of disturbance initially applied to the water and whether it is continuously applied to produce a forced wave or is quickly removed to allow the wave to propagate away as a free wave.
- The type of restoring mechanism that forces the water back to equilibrium.
- The properties of the wave itself.

These characteristics can be expressed in terms of the following parameters, shown in Figure 1.15:

- *Wavelength* (L) - distance from crest to crest or trough to trough.
- *Period* (T) - time between two successive crests or troughs passing a fixed point.
- *Amplitude* (A) - the maximum displacement of a wave.
- *Wave height* (H) - the vertical distance from trough to crest ($2 \times A$).
- *Depth of water* (D) - distance between mean surface level and sea bottom.
- *Speed* (celerity C) - speed at which a wave passes a fixed point.
- *Surface elevation* (η) - elevation of wave above mean sea level.

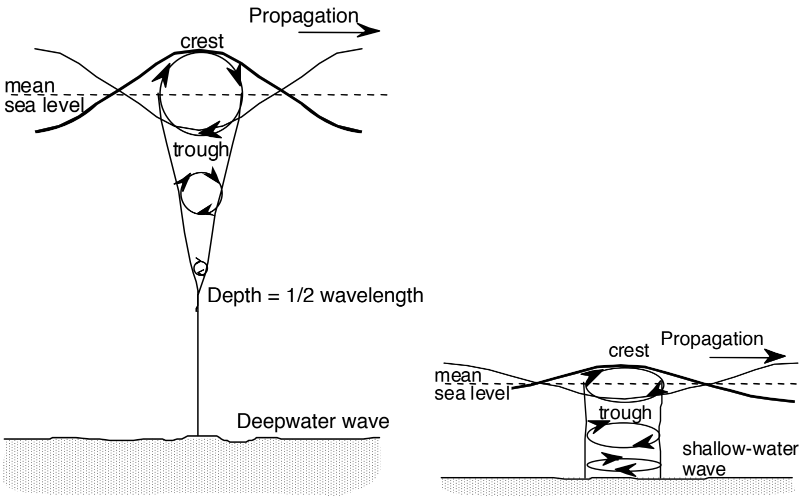

Figure 1.16: Water motion for deep (left) and shallow water waves.

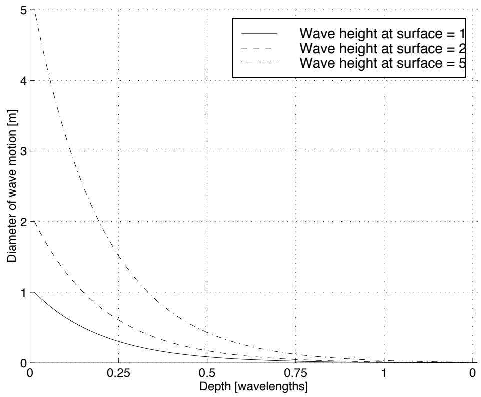

Figure 1.17: Short wave motion as a function of water depth.

The depth range valid for each type of wave is presented in Table 1.3. Deep water waves (short waves) are usually wind-generated and are unaffected by the sea bottom. Shallow waves (long waves) e.g., tide waves or tsunamis have wavelengths which exceed hundreds of kilometres whereas the ocean depth is less than 13 km.
Water does not move at the wave speed but according to a circular (deep water wave) or an elliptic motion (shallow water wave), as shown in Figure 1.16. For deep water waves, the motion is a combination of horizontal motion of the water (forth and back) with the vertical motion of the wave (up and down) which results in a nearly circular motion. Water motion decreases with depth. For short waves, the diameter of circular motion is described by

$$\mathrm{Diam}(z) = H \cdot \exp(2\pi \cdot z / L) \tag{1.8}$$

where Diam is the diameter of motion, H the height of the wave at the surface ($z = 0$), z the depth relative to the mean water level ($z \leq 0$) and L the wavelength. Since z is negative, Diam decreases rapidly as depth increases, see Figure 1.17, and the water becomes calm very rapidly as depth increases. The diameter of the circle for a deep water wave is $1/23H$ at a depth of $1/2L$. At a depth of L, it is only $1/535H$. This is why water gets calmer quickly as the depth increases. For intermediate water waves the circles become flattened into ellipses as the bottom restricts the vertical motion of water. For shallow water waves, the ellipses practically flatten to straight lines. The motions are almost entirely in a horizontal plane. The horizontal motions decrease only slightly from top to bottom.
There is another useful classification of waves in terms of period (or frequency). Spectral analysis is a convenient tool to represent the *wave energy* against the wave period (or frequency). The surface elevation η (vertical displacement of the free surface from the mean sea level) observed over time can be expressed as a trigonometric series

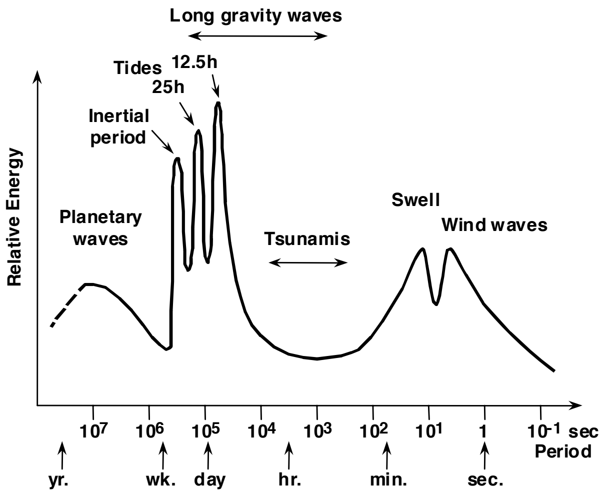

Figure 1.18: Wave energy spectrum versus period.

$$\eta = \sum_i A_i \{\sin(\omega_i t + \phi_i) + \cos(\omega_i t + \phi_i)\} \tag{1.9}$$

where A_i is the amplitude, ϕ_i is the phase, t is time and ω_i is the angular frequency in radians per second. Spectral analysis provides the amplitude and phase coefficients A_i and ϕ_i determined as a function of frequency *f*. A_i^2 is not only proportional to the wave energy, it is also proportional to the square of surface elevation η^2 and as such has units of m^2. A_i^2 versus frequency or period gives the wave *energy spectrum*, as shown in Figure 1.18. The basic types of surface waves are given in Table 1.4.

The theory of wave speed discussed below corresponds to waves of small

Table 1.4: Basic types of surface waves with approximate range of periods and wavelengths and primary generating and restoring mechanisms.

Name	**Period**	**Wavelength**	**Generating mechanism**	**Restoring mechanism**
Capillary waves (ripples, wavelets)	Less than 0.1 s	Less than 2 cm fluctuations	Wind, pressure, tension	Surface
Gravity waves (chop, sea, swell)	0.5 - 30 s	10 cm - 1000 m	Wind	Gravity
Infragravity waves	Minutes	100's of m - 100's of km	Storm systems (winds and atmospheric pressure gradients)	Gravity
Tsunamis	10's of minutes - 1 h	100's of km	Submarine earthquakes, shoreline slumping	Gravity
Tides	Mainly 12.5 and 25 h	1000's of km	Gravitational attraction of Sun and Moon	Gravity and Coriolis force

amplitudes. For a relative height or steepness, defined as H/L, less than 1/20, linear theory can be applied. For example, for a swell of length 200 m, the small amplitude theory would apply to a height of 10 m (which is very large for a given swell). The relative height in this case will be 10/200 = 1/20, which is the limit of the linear theory.
One of the most useful measures of larger waves is the *significant wave height* H_s which is defined as mean height of the highest one-third of the waves. The use of the significant wave height as a sea state forecast tool was developed during World War II. It is used in offshore engineering to calculate possible destructive effects on ship and drilling platforms. Assume a wave travelling in the *x*-direction. The vertical displacement η of the free surface from the mean level is

$$\eta = A\cos\{2\pi(\frac{x}{L} - \frac{t}{T})\} \tag{1.10}$$

where x is the actual displacement of the wave, t is time and T is the period. Using

$k = \frac{2\pi}{L}$, the radian wave number

$\omega = \frac{2\pi}{T}$, the radian (angular) frequency

the vertical displacement η can be expressed as

$$\eta = A\cos(kx - \omega t) \tag{1.11}$$

where $kx - \omega t$ is the *phase of the wave* which varies from 0 to 2π as one goes from one crest to the next (distance L). The wave speed C, defined as

$$C = \frac{L}{T} \tag{1.12}$$

is the speed at which a point of fixed phase travels. It is called a *phase speed*. The wave phase speed can be derived from the equation of motion for waves

$$C = \sqrt{\frac{gL}{2\pi}\tanh(\frac{2\pi h}{L})} = \sqrt{\frac{g}{k}\tanh(kh)} \tag{1.13}$$

where h is the depth of water below mean sea level and g is gravity.
If $L < 2h$, the wave is a short wave (found in deep water) and the above formula (1.13) is approximated by

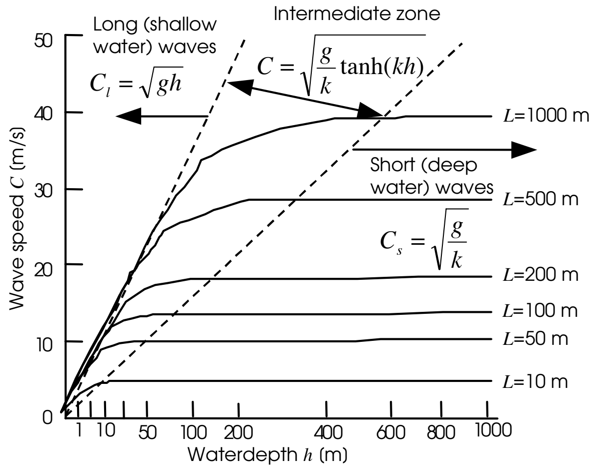

Figure 1.19: Wave speed versus water depth for various wavelenghts from 10 m to 1,000 m.

$$C_s = \sqrt{\frac{gL}{2\pi}} = \sqrt{\frac{g}{k}} \tag{1.14}$$

If $L > 2h$, the wave is a long wave (found in shallow water) and the formula is approximated by

$$C_l = \sqrt{gh} \tag{1.15}$$

The relationship between the radian frequency ω and radian wave number k, or equivalently between the wavelength L and period T is called the dispersion relation and is derived from the equation of motion. It has the following form

$$\omega^2 = gk\tanh(kh) \tag{1.16}$$

The wave speed as a function of water depth for wavelengths of 10 m to 1,000 m is shown in Figure 1.19.
The rate at which the energy of the wave is propagated is the group speed C_g; it can be expressed as

$$C_g = \frac{C_p}{2}\{1 + \frac{2kh}{\sinh(2kh)}\} \tag{1.17}$$

where C_p is the phase speed (C_s or C_l) and h is the depth of the wave. For long waves, $kh << 1$, $\sinh 2kh$ can be approximated by $2kh$ and $C_g \approx C_l$. Similarly, for short waves, $kh >> 1$, $\sinh(2kh) >> 2kh$ and $C_g \approx C_s / 2$. Since modulations are

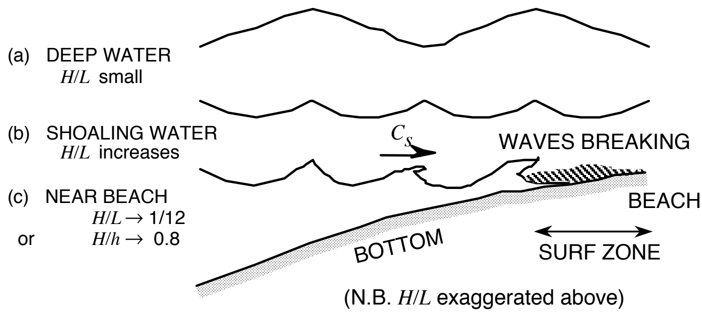

Figure 1.20: Transition of waves from deep to shallow water.

simply modifications of energy and may contain information, it follows that information can be transmitted no faster than the group speed. When the wave moves from deep to shallow waters, the period T remains constant, but C and L decrease, as shown in Figure 1.20.

Waves form on the sea surface by the transfer of energy from the air to the water. Amongst the different mechanisms generating waves, winds contribute most to the ruffled veneer of the waters on Earth, see Figure 1.21. The longer and harder the wind blows, the greater the amount of energy transferred from the wind to the waves. Wave energy is proportional to wave height. Wind waves are initiated by the wind blowing for some hours over a sea surface many kilometres long. The length of the sea surface directly affected by the wind is called the fetch. The oscillations of the sea surface caused by the wind continue to run across the sea far beyond the direct influence of the wind. The wind waves transferred beyond the fetch are called swell. The swell consists of fairly uniform wave trains. It decays for a long distance while its wavelength increases and wave height H decreases. When the swell enters shallow water:

- The wave speed C decreases.

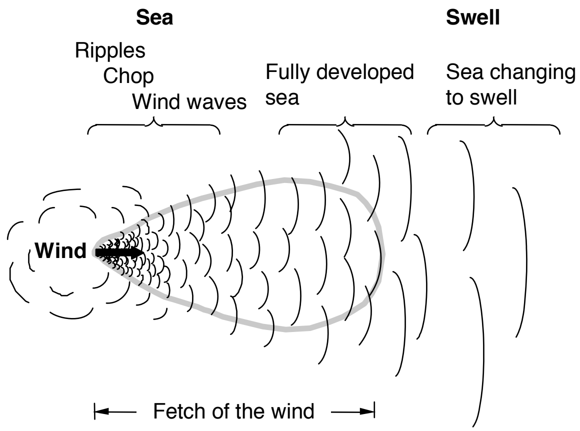

Figure 1.21: Wind generated waves.

Table 1.5: Minimum fetch and duration to produce fully developed seas at various wind speeds. Significant height is average height of highest 1/3 of observed waves, maximum probable wave height is 1.8 × significant height.

Wind speed [m/s]	Fetch [km]	Duration [h]	Average height [m]	Significant height [m]	Average highest 10% waves [m]	Period of greatest energy concentration [s]
5	19	2.4	0.3	0.4	0.6	4
8	63	6	0.8	1.1	1.5	6
11	139	10	1.5	2.4	3.1	8
13	296	16	2.7	4.3	5.5	10
16	518	23	4.3	6.7	8.4	12
22	1315	42	8.5	13.4	17.4	16
27	2630	69	14.6	23.8	30.2	20

- The wavelength L decreases.
- The wave height H increases.
- The period T remains constant.

The swell finally peaks up into waves, breaks, and is dissipated as surf. Wind can saturate the sea with energy, and waves begin to break and result to a fully developed sea. This process is a function of wind speed, fetch and duration. For a fully developed sea, the *duration, fetch* and *significant wave height* H_s are given in Table 1.5.

The *Beaufort Scale* of wind force, originally devised by Admiral Beaufort in the early 19th century, relates wind speed to the physical appearance of sea surface by considering an apparent wave height, the prominence of breakers, whitecaps, foam and spray, see Table 1.6. It has since been repeatedly modified to make it more relevant to modern navigation.

Information concerning the wave heights and periods can be obtained using a number of methods, namely

- Visual method - based on a visual estimate of the sea state.
- Vertical scale method - based on visual observation of the water surface against a vertical scale mounted on a pier or on a float equipped with a deep horizontal damper to limit vertical movement.
- Pressure sensor method - based on the measurement of the hydrostatic pressure below surface waves (the pressure varies as a function of water depth).
  - Pressure sensor mounted below depth of deepest trough; practical only for deep waters (short waves).
  - Bottom-mounted pressure sensors for long waves (shallow waters) such as tsunamis and tides.
  - On a ship, the pressure sensor is mounted on the lower part of the ship's hull; a one-axis (vertical) accelerometer is used to measure vertical motion. The differences in wave heights are determined.
- Electrical method - based on the electrical penetration of the sea surface. Many pairs of wires, with a small gap between the wires of each pair, are mounted horizontally on vertical rods. Immersed wires are short circuited and, by

Table 1.6: Beaufort wind scale.

Beaufort number	Wind	Sea state	Wind speed [km/hr]	Wave height [m]
0	Calm	Sea smooth as mirror	< 1	0
1	Light air	Small wavelet-like scales; no foam crests	2-5	0.15
2	Light breeze	Waves short; crests begin to break	6-11	0.3
3	Gentle breeze	Foam has glassy appearance; not yet white	12-20	0.6
4	Moderate breeze	Waves now longer; many white areas	21-29	1.6
5	Fresh breeze	Waves pronounced and long; white foam crests	30-39	3.1
6	Strong breeze	Larger waves form; white foam crests all over	40-50	4.7
7	Moderate gale	Sea heaps up; wind blows foam instreaks	51-61	6.2
8	Fresh gale	Height of waves and crests increasing	62-74	7.8
9	Strong gale	Foam is blown in dense streaks	75-87	9.3
10	Whole gale	High waves with long overhanging crests; large foam patches	88-101	10.8
11	Storm	High waves; ships in sight hidden in troughs	102-120	-
12	Hurricane	Sea covered with streaky foam; air filled with spray	> 120	-

recording continuously the number of pairs from the bottom which are short-circuited, a record of sea surface level is obtained. This method is used for analysis of detailed wave structures.

- Airborne and satellite-borne radar altimetry - based on measuring distance from the aircraft or the satellite to the sea surface. From the observations and known positions of the aircraft or satellite the surface elevation along the flight path can be derived.

Internal waves may occur in underwater surfaces when vertical density variations are present (sea consists of layers of different density). Density differences between the layers result in restoring forces (gravitational or hydrostatic pressure). They are caused by, e.g., current shear or surface disturbances. Internal waves are frequent in oceanic waters on the thermocline (layer at a depth of 50 - 100m with large temperature variations) and in coastal waters on the halocline (layer with large salinity variations).

1.4 Major references

Apel, J.R.: *Principles of ocean physics.* International Geophysics Series, Vol. 38, Academic Press, 1987.

Lutgens, F.K., E.J. Tarbuck: *The atmosphere - an introduction to meteorology*. Prentice Hall, 1989.

Neumann, G, W.J. Pierson, Jr.: *Principles of physical oceanography*. Prentice Hall, 1966.

Pond, S., G.L. Pickard: *Introductory dynamical oceanography*. Pergamon Press, 1983.

Thomson, R.E.: *Oceanography of the British Columbia coast*. Fisheries and Aquatic Sciences 56, Fisheries and Oceans, Canada, 1981.

Van Dorn, W. G.: *Oceanography and seamanship*. Cornell Maritime Press, 1993.