

# ***Transport Phenomena***

## ***The Art of Balancing***

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# Preface

In 1956, Hans Kramers in Delft published his first lecture notes (in Dutch) on ‘Fysische Transportverschijnselen’ – to the best of our knowledge the first systematic treatment of the emerging discipline of Transport Phenomena. In 1958, Robert B. Bird spent a period in Delft as a guest of Hans Kramers. This visit gave the two Professors the excellent opportunity to explore and improve the way of teaching Transport Phenomena. Bird published his ‘Notes on Transport Phenomena’ in the Fall of 1958, followed in 1960 by the first Wiley edition of the famous ‘Transport Phenomena’ textbook by Bird, Stewart & Lightfoot.

In Delft, the Dutch students kept using Kramers’ shorter lecture notes in Dutch which in the course of the years were continuously improved, also by Kramers’ successors Wiero J. Beek and John M. Smith. All those years the analogy of momentum, heat and mass transport remained the leading theme, just like in Bird’s textbook. An essential element in the way Transport Phenomena has been taught in Delft has always been the emphasis on developing the students’ ability of solving realistic engineering problems. Over the years, hundreds of challenging exam problems were devised.

In 1996, the current authors published a new version of the Delft lecture notes on Transport Phenomena for various reasons. Students, their interests, their prior education, and the way they prepare for exams were changing. In many curricula, the course got a different role and place. New applications in biotechnology, biomedical, smart materials and solar developed – remote from the traditional chemical industry. And Computational Fluid Dynamics (CFD) developed into a real analytical tool. All this required different didactic methods for teaching as well as different examples and exam problems.

Our new version of the Delft textbook ‘Transport Phenomena’ (still in Dutch) built on the earlier Delft lecture notes but was still based on the classical analogy of momentum, heat and mass transport, although the order of treatment was changed: fluid mechanics largely moved to the end, provoked by ideas developed by Kees Rietema at Eindhoven University of Technology. Most importantly, however, we put a much stronger emphasis on the basic method of drawing up balances, either about a particular device (a macro-balance) or about a differential element anywhere in a material or fluid (a micro-balance). In most cases, such a balance turns into a differential equation. We believe that teaching students as to how to draw up balances and solve differential equations is an excellent preparation for exploiting

modern CFD techniques. The exam requirement that students should be capable of solving original problems was maintained.

Our textbook was quite successful: a 2<sup>nd</sup> edition was released in 2003-2005, a 3<sup>rd</sup> edition in 2008. In recent years, however, increasing numbers of foreign students arrived at Delft University of Technology for various MSc programs. This development has prompted the idea of publishing an English version of our Dutch textbook, simultaneously updating and improving a Dutch 4<sup>th</sup> edition. This textbook – Transport Phenomena: The Art of Balancing – is the result. We hope it will find its way to foreign universities as well.

We like to express our sincere appreciation for all suggestions for improvements we received over the years. In particular we like to acknowledge the contributions from A.G.N. Boers, C. Ouwerkerk, G.C.J. Bart, C.R. Kleijn, J.J.Ph. Elich, L.M. Portela, J.A. Battjes, R.B. Bird, and J.E. Schievink. We remain open for suggestions of further improvements.

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# 1 Balances

## 1.1 The balance: recipe and form

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The field of transport phenomena covers the transport of the three most important quantities – mass, energy, and momentum – in any (physical or chemical) process. The addition of the words ‘in any process’ in particular is an indication of one of the most important features of the field: transport phenomena is, above all, an engineering field with a wide range of applications.

Nonetheless, the field is also fundamental, given that it forms the basis for many other chemical engineering disciplines, such as reactor engineering, separation technology, and fluid mechanics. This makes transport phenomena a must for any chemical engineer. A good knowledge of the subject is also very useful to those in other professions, such as mechanical, mining, civil, and building engineers, physicists, chemists, and materials scientists.

The area covered by the field of transport phenomena and the discipline of chemical engineering is considerable. There are, for example, all kinds of processes in the chemical and petro-chemical industry, the flow of one or more phases through a pipeline, the behaviour of bubbles in a bioreactor, or the filling of a casting mould with liquid metal. At the other end of the scale, the field is also very important to more everyday matters, such as the heat emission of a radiator and the associated air flows in the room, and transport of oxygen by blood flow. Fortunately, these very different processes can be clearly understood and described with a limited number of rules.

Flow phenomena and heat and mass transfer are described in this field in terms of continuum properties, with only occasional references to molecular processes. This is how the basis is laid for chemical engineering: the expertise of designing and improving processes in which substances are transported, transformed, processed, or shaped. It is important here to fully understand the essence of a process – that is, to identify the essential stages in the transport of mass, heat, and/or momentum. The transport of these three quantities can, as it happens, be described in exactly the same way. Transport phenomena lays the basis for physical technology and provides the necessary tools. This textbook is about these tools.

First and foremost, transport phenomena is a subject of *balances* and *concepts* by which physical processes and phenomena can be described. In many cases, the subject is about deviations from a state of equilibrium and the subsequently occurring *resistances* to heat and mass transport. It frequently concerns a quantitative description of cause and effect. With the help of these still somewhat vague terms, it is possible to gain an outlined, but also very detailed, understanding and description of the aforementioned and countless other processes. This chapter will discuss the term *balance* in extensive detail.

For the description of the transport of any quantity, such as the transport of oxygen from bubbles to the liquid phase in a fermenter or the transport of heat through the wall of a furnace, the balance is an essential tool. The basic principle of the balance is the bookkeeping of a selected physical quantity. This concept is of particular importance when working with what are known as conserved quantities; these are quantities (like mass and energy) that are not lost during a process, but conserved.

The field of transport phenomena deals with steady-state or transient (time-dependent) processes in which mass, energy and momentum are exchanged between domains as a result of driving forces (differences in concentrations of mass, energy and momentum, and/or in pressure). Transport phenomena is therefore primarily about the ‘bookkeeping’ of the three physical quantities: mass, energy and momentum.

This bookkeeping can refer to large control volumes, which involve *macrobalances*; however, balances can also be drawn up in relation to very small control volumes – these are known as *microbalances*, which provide information at a local scale. In almost all cases, solving problems such as about transport or transfer rates, or about changes in concentrations or temperatures, starts with drawing up one or more balances.

The next step is to derive from such balances proper equations, in many cases differential equations; the latter require initial and/or boundary conditions. The final step is about solving these (differential) equations to find the answer to the problem under consideration. In this approach, it is essential to denote all quantities with symbols!

The *general recipe* for drawing up a balance and solving the problem can be summarised as follows:

- 1) Make a sketch of the situation. Use symbols rather than numerical values to indicate quantities.
- 2) Select the quantity  $G$  that is being transported or transferred in the process under consideration.
- 3) Select the ‘control volume’  $V$  about which information is to be obtained.

- 4) Find out whether and if so, how, the quantity of  $G$  in the control volume  $V$  changes during a brief period of time  $\Delta t$ . Draw up the balance (using symbols).
- 5) Solve the (differential) equation resulting from the balance.

The quantity of  $G$  in  $V$  can change in all kinds of ways. These should be examined systematically and, if applicable, included in the balance. For example, during  $\Delta t$ ,  $G$  can flow into  $V$  from outside. As a result, the quantity of  $G$  inside  $V$  increases. It is also possible for  $G$  to flow outwards, from inside  $V$ . In this case, the quantity of  $G$  in  $V$  falls. We refer to “inflow” and “outflow”, respectively. Of course, it is also possible for *production* of  $G$  to occur inside  $V$  during period  $\Delta t$ : as a result, the total quantity of  $G$  in  $V$  increases. Negative production (= destruction, consumption, annihilation) is also possible, for example if  $G$  stands for the mass of a reagent that is being transformed in a chemical process.

Bear in mind that  $G$  may not necessarily be the quantity in which you are interested. In order to calculate temperature  $T$ , for example, a thermal energy balance has to be drawn up, and the thermal energy  $U$  must be selected for  $G$ .

The general structure for a balance is now as follows (see also Figure 1.1):

$$\begin{aligned}
 & \text{The change of } G \text{ in } V \text{ during } \Delta t = \\
 & = G \text{ (at time } t + \Delta t \text{) in } V - G \text{ (at time } t \text{) in } V \\
 & = \text{ quantity of } G \text{ that flows from outside into } V \text{ during } \Delta t + \\
 & \quad - \text{ quantity of } G \text{ that flows outside from inside } V \text{ during } \Delta t + \\
 & \quad + \text{ net quantity of } G \text{ that is produced in } V \text{ during } \Delta t
 \end{aligned}$$

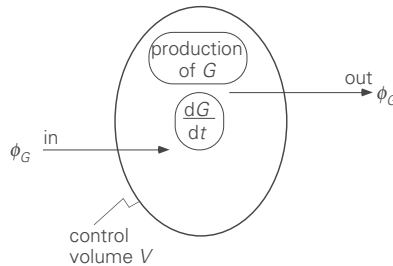


Figure 1.1.

From now on, the symbol  $\phi$  will be used to denote a transport (rate), with the dimension ‘quantity of  $G$  per unit of time’. Instead of transport rate, the term *flow rate* is used. The letter  $P$  stands for net production per unit of time, or net production rate. With the help of this notation, the quantity of  $G$  that flows ‘inwards’ (= from the outside to the inside) *during* the period of time  $\Delta t$  can, if  $\Delta t$  is very short, be written as the product of the flow rate ‘in’ *at time*  $t$  and the period of time  $\Delta t$ :