

ELECTRONIC INSTRUMENTATION

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© **Delft Academic Press**

Third edition 2015

Published by:

Delft Academic Press /VSSD

Leeghwaterstraat 42, 2628 CA Delft, The Netherlands

tel. +31 15 27 82124

dap@vssd.nl

www.delftacademicpress.nl

www.delftacademicpress.nl/e008.php

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Printed in The Netherlands.

ISBN 97890-6562-3799

NUR 959

Keywords: electronic instrumentation

Preface

Electronic systems have made deep inroads into every aspect of daily life. One need only look around homes, offices and industrial plants to see that they feature almost everywhere. Indeed, it is practically impossible to name any appliances, tools or instruments that do not contain electronic components. In order to compete with rival companies or just remain a step ahead of them, the designers of technical systems and innovative products must be fully aware of both the assets and the limitations of electronic components and systems. Users of electronic systems also need to have a basic knowledge of electronic principles. In order to fully exploit an instrument's potential, to be aware of its limitations, to correctly interpret the measurement results and to be able to arrive at well-balanced decisions relating to the purchasing, repairing, expansion or replacement of electronic equipment, all users of such systems also need to have a basic knowledge of electronic principles.

This book offers such basic knowledge and provides guidance on how to obtain the relevant skills. The kinds of topics dealt with are operating principles, the performance of analog and digital components and circuits, and the precise characteristics of electronic measuring systems. Throughout the book, every endeavor is made to impart a critical attitude to the way in which such instruments should be implemented.

The book is based on various series of courses on electronics and electronic instrumentation that were given by the author during the many years when he lectured at Delft University of Technology in the Netherlands. The courses were designed for students from various departments such as: Mechanical Engineering, Aeronautical Engineering and Mining Engineering. When numbers of non-Dutch-speaking Master of Science students started to rise it became necessary to publish an English version of the book.

The particular way in which the book has been organized makes it suitable for a much wider readership. To meet the demands of divergent groups it has been structured in a modular fashion. Each chapter discusses just one particular topic and is divided into two parts: the first part provides the basic principles while more specific information is given in the second part. Each chapter ends with a summary and several exercises. Answers to all the exercises are given at the back of the book. This approach is conducive to self-study and to the composition of tailor-made course programs.

The required background knowledge is a basic grounding in mathematics and physics equivalent to any first-year academic level. No background knowledge of electronics is needed to understand the contents of the book. For further information on particular

subjects the reader is referred to the many course books that exist on the subjects of electronics, measurement techniques and instrumentation.

I am indebted to all the people who contributed to the realization of this book. In particular I would like to thank Johan van Dijk who carefully refereed the original Dutch text. I am grateful also to Reinier Bosman for working out all the exercises, to G. van Berkel for creating the more than 600 illustrations, to Jacques Schievink for processing the original Dutch editions and this English version of the book and to Diane Butterman for reviewing the entire English text.

Paul Regtien
Hengelo, August 2004

Preface to the third edition 2015

In this edition several corrections have been made to the second edition.

The publisher
Delft, september 2015

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1 Measurement systems

The aim of any *measuring system* is to obtain information about a physical process and to find appropriate ways of presenting that information to an observer or to other technical systems. With electronic measuring systems the various instrument functions are realized by means of electronic components.

Various basic system functions will be introduced in the first part of this chapter. The extent to which an instrument meets the specified requirements is indicated by the system specifications, all of which will be discussed in the second part of the chapter.

1.1 System functions

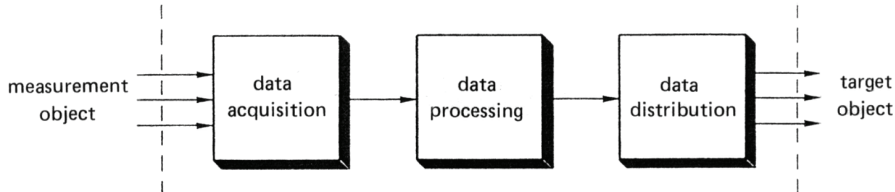


Figure 1.1. The three main functions of any measuring system.

- *Data acquisition*: this involves acquiring information about the measurement object and converting it into electrical measurement data. What multiple input, as illustrated in Figure 1.1, indicates is that invariably more than one phenomenon may be measured or that different measurements may be made, at different points, simultaneously. Where there are single data outputs this means that all data is transferred to the next block through a single connection.
- *Data processing*: this involves the processing, selecting or manipulating – in some other way – of measurement data according to a prescribed program. Often a processor or a computer is used to perform this function.
- *Data distribution*: the supplying of measurement data to the target object. If there is multiple output then several target instruments may possibly be present, such as a series of control valves in a process installation.

It should be pointed out that the above subdivision cannot always be made; part of the system may sometimes be classified as both data acquisition and data processing. Some authors call the entire system shown in Figure 1.1 a data acquisition system, claiming that the data is not obtained until the target object is reached.

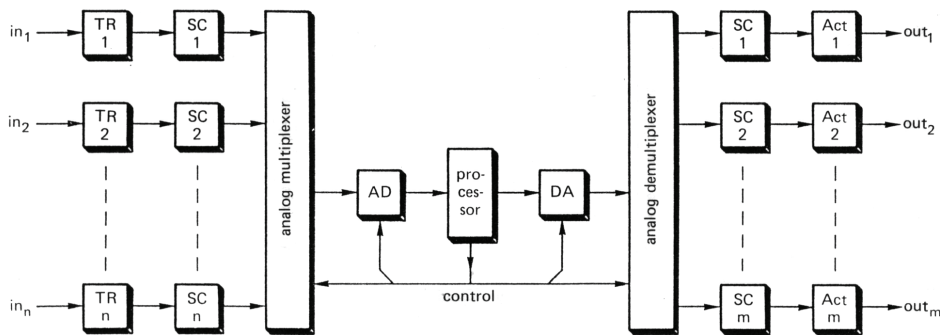


Figure 1.4. A multi-channel measuring system with a centralized processor and AD and DA-converters. For an explanation of the abbreviations see Figure 1.3.

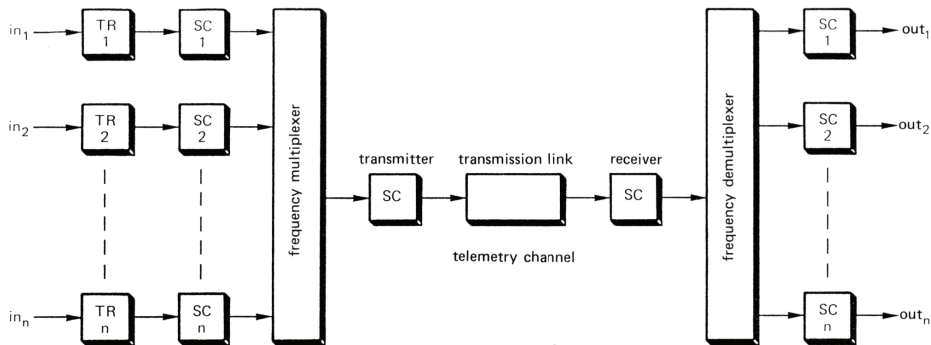


Figure 1.5. A multi-channel measuring system with frequency multiplexing.

Signals can be transmitted in analog or digital form. Digital transport is preferable if high noise immunity is required, for instance for very long transport channels or links that pass through a noisy environment.

1.2 System specifications

A measurement system is designed to perform measurements according to the relevant specifications. Such specifications convey to the user of the instrument to what degree the output corresponds with the input. The specifications reflect the quality of the system.

The system will function correctly if it meets the specifications given by the manufacturer. If that is not the case it will fail, even if the system is still functioning in the technical sense. Any measuring instrument and any subsystem accessible to the user has to be fully specified. Unfortunately, many specifications lack clarity and completeness.

The input signal of the single channel system given in Figure 1.6 is denoted as x and its output signal as y . The relationship between x and y is called the system transfer.

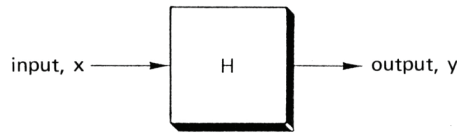


Figure 1.6. Characterization of a system with input x , output y and transfer H .

By observing the output, the user is able to draw conclusions about the input. The user therefore has to be completely familiar with the system's transfer. Deviations in the transfer may cause uncertainties about the input and so result in measurement errors. Such deviations are permitted, but only within certain limits which are the tolerances of the system. Those tolerances also constitute part of the specifications. In the following pages the main specifications of a measurement system will be discussed.

The user should first of all be familiar with the operating range of the system. The operating range includes the measurement range, the required supply voltage, the environmental conditions and possibly other parameters.

Example 1.1

A manufacturer of a digital temperature-measuring instrument gives the following description of the operating range:

- * *measuring range: -50°C to 200°C ;*
- * *permitted operational temperature: -10°C to 40°C ;*
- * *storage temperature: -20°C to 85°C*
- * *mains voltage: $220\text{ V} \pm 15\%$, $50\ldots 60\text{ Hz}$; can be switched to 115 V , 127 V , $240\text{ V} \pm 15\%$, $50\ldots 60\text{ Hz}$;*
- * *analog outputs: $0\text{--}10\text{ V}$ (load $> 2\text{ k}\Omega$) and $0\text{--}20\text{ mA}$ or $4\text{--}20\text{ mA}$ (load $< 600\Omega$).*

All other specifications only apply under the condition that the system has never before been taken beyond its permitted operating range.

The resolution indicates the smallest detectable change in input quantity. Many system parts show limited resolution. A few examples of this are these: a wire-wound potentiometer for the measurement of angles has a resolution set by the windings of the helix – the resistance between the slider and the helix changes leap-wise as it rotates; a display presenting a measurement value in numerals has a resolution equal to the least significant digit.

The resolution is expressed as the smallest detectable change in the input variable: Δx_{\min} . Sometimes this parameter is related to the maximum value x_{\max} that can be processed, the so-called full-scale value or FS of the instrument, resulting in the resolution expressed as $\Delta x_{\min}/x_{\max}$ or $x_{\max}/\Delta x_{\min}$. This mixed use of definitions seems very confusing. However, it is easy to see from the units or the value itself which definition is used.

Example 1.2

The resolution of a four-digit decimal display with a fixed decimal point in the third position from the left is 0.1 units. The maximum indication apparently equals 999.9 units, which is about 1000 . The resolution of this display is therefore 0.1 units or 10^{-4} or 10^4 .

The inaccuracy is a measure of the total uncertainty of the measurement result that may be caused by all kinds of system errors. It comprises calibration errors, long and short-term instability, component tolerances and other uncertainties that are not separately

specified. Two definitions may be distinguished: absolute inaccuracy and relative inaccuracy. Absolute inaccuracy is expressed in terms of units of the measuring quantity concerned, or as a fraction of the full-scale value. Relative inaccuracy relates the error to the actual measuring value.

Example 1.3

The data sheet of a volt meter with a four digit indicator and a full-scale value of 1.999 V specifies the instrument inaccuracy as $\pm 0.05\%$ FS $\pm 0.1\%$ of the indication $\pm \frac{1}{2}$ digit.

The absolute inaccuracy of a voltage of 1.036 V measured with this instrument equals: ± 0.05 of 2 V (the approximate value of FS) plus $\pm 0.1\%$ of 1 V (approximate value of the indication) plus ± 0.5 of 1 mV (the weight of the last digit), which amounts to ± 2.5 mV in total.

The relative inaccuracy is the absolute inaccuracy divided by the indication so it is $\pm 0.25\%$.

Inaccuracy is often confused with accuracy, the latter being complementary to it. When a specification list gives an accuracy of 1%, this hopefully means that there is an inaccuracy of 1% or an accuracy of 99%.

The sensitivity of a measuring system is defined as the ratio between a change in the output value and a change in the input value that causes that same output change. The sensitivity of a current-to-voltage converter is expressed in V/A, that of a linear position sensor in, for instance, mV/ μ m and that of an oscilloscope in, for instance, cm/V.

A measuring system is usually also sensitive to changes in quantities other than the intended input quantity, such as the ambient temperature or the supply voltage. These unwelcome sensitivities should be specified as well when this is necessary for a proper interpretation of the measurement result. To gain better insight into the effect of such false sensitivity it will be related to the sensitivity to the measurement quantity itself.

Example 1.4

A displacement sensor with voltage output has a sensitivity of 10 mV/mm. Its temperature sensitivity is -0.1 mV/K. Since -0.1 mV corresponds with a displacement of -10 μ m, the temperature sensitivity can also be expressed as -10 μ m/K. A temperature rise of 5°C will result in an apparent displacement of -50 μ m.

Example 1.5

The sensitivity of a temperature sensor including the signal-conditioning unit is 100 mV/K. The signal conditioning part itself is also sensitive to (ambient) temperature and it appears to create an extra output voltage of 0.5 mV for each °C rise in ambient temperature (not necessarily the sensor temperature). The undesired temperature sensitivity is thus 0.5 mV/K or $0.5/100 = 5$ mK/K. A change in ambient temperature of $\pm 10^\circ\text{C}$ gives an apparent change in sensor temperature that is equal to ± 50 mK.

Mathematically, the sensitivity is expressed as $S = dy/dx$. If output y is a linear function of input x then the sensitivity does not depend on x . In the case of a non-linear transfer function $y = f(x)$, S will depend on the input or output value (Figure 1.7). Users of measuring instruments prefer a linear response, because then the sensitivity can be

expressed in terms of a single parameter and the output will not show harmonic distortion. The transfer of a system with slight non-linearity may be approximated by a straight line. The user should still know the deviation from the actual transfer as specified by the non-linearity.

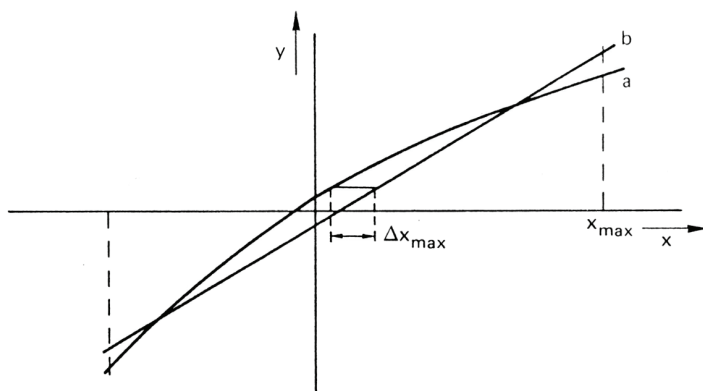


Figure 1.7. Example of a non-linear transfer characteristic, (a) real transfer, (b) linear approximation.

The non-linearity of a system is the maximum deviation in the actual transfer characteristic from a pre-described straight line. Manufacturers specify non-linearity in various ways, for instance, as the deviation in input or output units: Δx_{\max} or Δy_{\max} , or as a fraction of FS: $\Delta x_{\max}/x_{\max}$. They may use different settings for the straight line: by passing through the end points of the characteristic, by taking the tangent through the point $x = 0$, or by using the best-fit (least-squares) line, to mention but a few possibilities.

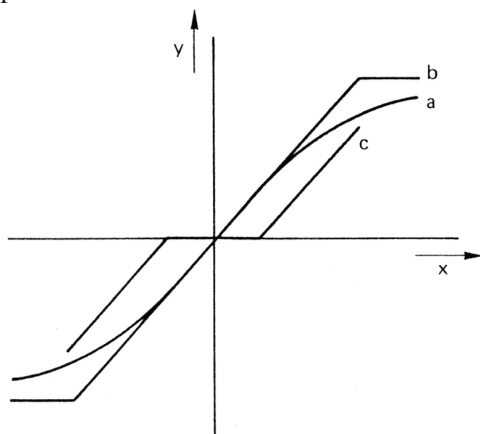


Figure 1.8. Some types of static non-linearity: (a) saturation, (b) clipping. (c) dead zone

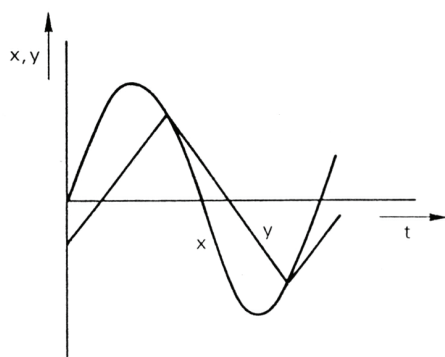


Figure 1.9. The effect of slew rate limitation on the output signal y at a sinusoidal input x .

Figure 1.8 depicts some particular types of non-linearity found in measuring systems: saturation, clipping and dead zone (sometimes also called cross-over distortion). These are examples of static non-linearity, appearing even when inputs change slowly. Figure

2 Signals

Physical quantities that contain detectable messages are known as signals. The information carrier in any electrical signal is a voltage, a current, a charge or some other kind of electric parameter.

The message contained in such a signal may constitute the result of a measurement but it can also be an instruction or a location code (like, for instance, the address of a memory location). The nature of the message cannot be deduced from its appearance. The processing techniques for electronic signals are as they are, regardless of the contents or nature of the message.

The first part of this chapter will concentrate on the characterization of signals and the various values of signals in terms of time functions. Signals may alternatively be characterized according to their frequency spectrum. In the case of periodic signals, the frequency spectrum is determined by means of Fourier expansion.

The second part of this chapter deals with aperiodic signals, in particular: noise, stochastic and sampled signals.

2.1 Periodic signals

2.1.1 A classification of signals

There are many ways to classify signals but one way is on the basis of their dynamic properties.

- Static or DC signals (DC = direct current, a term that is also applied to voltages): the signal value remains constant during the measuring time interval.
- Quasi-static signals: the signal value varies just a little, according to a given physical quantity. An example of a quasi-static signal is drift.
- Dynamic signals: the signal value varies significantly during the observation period. Such signals are also termed AC signals (AC = alternating current or alternating voltages).

Another way to distinguish signals is on the basis of the difference between deterministic and stochastic signals. What characterizes a stochastic signal is the fact that its exact value is impossible to predict. Most measurement signals and interference signals, such as noise, belong to this category. Examples of deterministic signals are:

- Periodic signals, characterized as $x(t) = x(t + nT)$, in which T is the time of a signal period and n the integer.

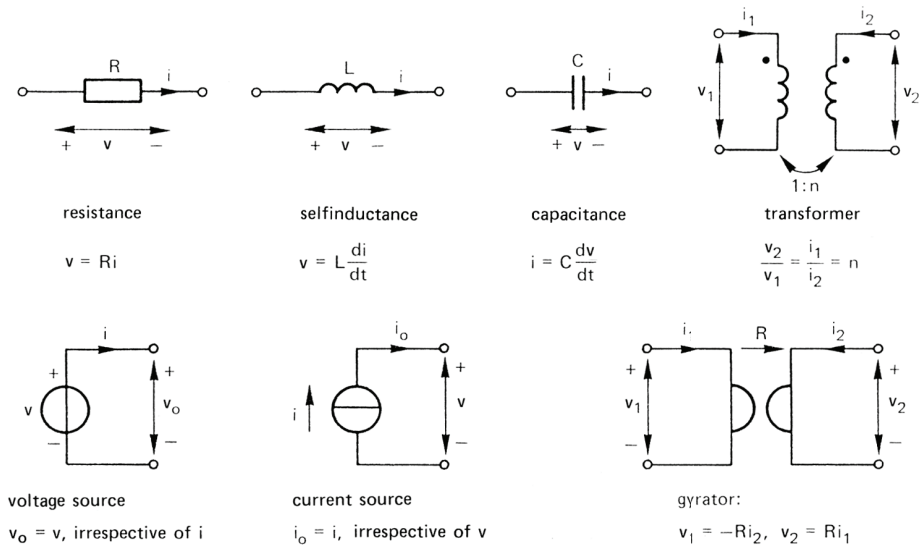
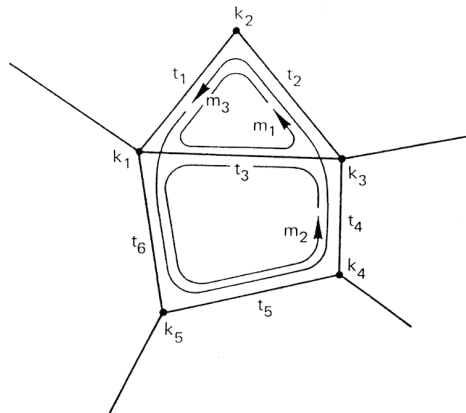


Figure 3.1. All electric network elements.

When modeling an electronic system composed of several components the corresponding network elements are connected to each other to form a complete network model of the system. A network contains nodes, branches and loops (Figure 3.2). A branch contains the network element, a node is the end of a branch and a loop is a closed path following arbitrary branches.



Within networks it is Kirchhoff's rules that apply:

- the rule for currents: the sum of all currents flowing towards a node is zero: $\sum_k i_k = 0$
- the rule for voltages: the sum of all voltages along a loop is zero: $\sum_m v_m = 0$

4 Mathematical tools

In order to compute the voltages and currents of an electric network one first has to solve a set of differential equations, an activity which – even with relatively simple networks – can be rather time-consuming. In this chapter two ways of facilitating the computations will be discussed. In the first part, we shall introduce the complex variables which can be used as a mathematical tool to calculate currents and voltages in a network without having to determine and solve the differential equations. The method is simple, but only valid for sinusoidal signals. In the second part, the Laplace transform, which can be used as a mathematical tool when computing arbitrary signals, will be introduced.

4.1 Complex variables

4.1.1 *The properties of complex variables*

This chapter commences with a brief overview of the main properties of complex variables. A complex variable is defined as the sum of a real variable and an imaginary variable. The latter is the product of a real variable and the imaginary unit $I = \sqrt{-1}$. To avoid confusion with the symbol i which stands for electric current, electrical engineers quickly adopted the symbol j instead: $j = \sqrt{-1}$. A complex variable is given as $z = a + jb$, in which a and b are real. The variables a and b are termed the real and imaginary components of the complex variable z . They are alternatively denoted as $\text{Re } z$ and $\text{Im } z$, respectively. A complex variable can therefore be presented as $z = \text{Re } z + j \text{Im } z$.

Real variables are presented as points on a straight line. Complex variables are presented as points in a complex plane, their coordinates being positioned on the real and imaginary axes (Figure 4.1).

This same figure provides another representation of z , using a length and an angle as the two coordinates (polar coordinates). The distance between z and the origin (0,0) is the modulus or absolute value of z which is denoted as $|z|$. The angle between the “vector” and the positive real axis is the argument z , denoted as $\arg z$, or simply given as a symbol for an angle, such as φ . The relationship between these two representations emanates directly from Figure 4.1:

$$|z| = \sqrt{(\text{Re } z)^2 + (\text{Im } z)^2} \quad (4.1)$$