## Preface

It has become quite a familiar image: people waiting in departure halls, airline passengers, and people on the beach, all of them hidden behind a booklet with squares containing numbers. The brand awareness of this new game, called sudoku, is high and almost everyone knows what you are talking about when you mention that you are solving a Sudoku puzzle. It has become enormously popular, young and old being engaged with it. There are simple puzzles for beginners, somewhat tougher ones for the more experienced, and extremely difficult and challenging puzzles for the real cracks. In addition, nowadays, there are quite a few variations on the original sudoku. Some contain extra grey areas that should also contain the numbers 1 through 9 ; others consist of two, three or five combined puzzles.

Sudoku originates from Japan¹, and is made up of the two Japanese words "so" and "doku". "So" means "number", while "doku" can be translated as "single". In other words, it is a puzzle with single numbers, but does this make it clearer? By the way, the official Japanese name for this kind of puzzles is much longer, namely "Suji wa dokushin ni kagiru", meaning that numbers are restricted to singles.

Owing to the fact that the Japanese language makes use of many letters, in fact pictograms, it is impossible to make crosswords, Mephistos or cryptograms in that language. This explains why the Japanese engage mainly in puzzles like Sudoku, Kakura, Hitori, Arukone, Filomino and many others. Often these puzzles consist of finding numbers or making pictures by thoroughly following hints and clues. There are many websites where one can find such puzzles; just type the keywords into a search engine. The websites of many newspapers publish daily Sudoku puzzles. Besides this there are also national and international sudoku organisations publishing a wealth of puzzles ranging from easy to extreme.

There are also many websites that offer online solutions to Sudoku puzzles. Using these one can enter the diagram of the original puzzle or of a partly solved puzzle for which one has already filled in a lot of numbers but got stuck at some point. One will then receive a hint as to how to proceed. Especially if one is at the stage of solving difficult or challenging puzzles, these websites can be quite convenient, although not all of them provide an adequate solution. In fact, many return the message "stuck" for the more difficult puzzles, hence leaving the puzzler in the dark.

[^0]Our favourite website is https://www.sudokuhints.com, which gives very good hints when one is at the level of Steps 1-8 in this book. Only when the last resort is needed, described in the chapter about Step 9, does this website fail to provide the answer. However, once the reader has made a choice of number pair, it can still be used to see whether this choice leads to an illegal Sudoku diagram and hence a contradiction. The best and most complete Sudoku solver is probably found on the website https://www.sudokuwiki.org, where many exotic techniques are used.

New developments are encountered all the time. If one owns a smartphone, one can download an application ("app") that enables the user to store a Sudoku by taking a photograph. One can also obtain hints and a full solution!

Finally the world wide web also contains a lot of information on solution strategies and techniques. It is striking, however, that most of the websites do not contain much detail. Often techniques are explained using just one example, and other exercises or worked examples are not given. In many cases the website appears to have been put together in a hasty way, or in such a way that the material is only accessible by experts. On the other hand, it is nice to observe the efforts concerning new patterns, with appealing names like jellyfish, swordfish or squirm bag.

The foregoing observations provide one of the main motives for compiling this book, which contains a thorough description of a sequence of techniques, together with many worked examples and ample exercises tailored to the techniques discussed. It is surprising that, despite the enormous popularity of Sudoku, there are still only a few accessible books on solution methods ${ }^{2}$. Often, techniques are discussed without much detail; in addition, there is not much material available for practicing. In most cases the thread of the story is missing, in the sense that one has a collection of techniques that are not at all related. A systematic approach is needed, and this is what is offered in this book.

Another goal in writing this book is to present a complete set of techniques, that is suitable for all contemporary puzzle magazines, and that leads to the solution in a guaranteed way! There are many different magazines presenting sudokus of different degrees of difficulty, but the reader will notice that the collection of techniques presented here will enable the full solution of each of the puzzles encountered. Our unique methodology is based entirely on number pairs, and this has proven to be an extremely powerful approach. It provides the key to solving sudokus, even those which are classified as 6 -star or even higher, sometimes also termed "challenging", "hard", "extreme", "wicked" or "fiendish".

[^1]A big advantage of the methodology presented in this book is that one may use a pen most of the time, because one can be certain that the numbers being filled in are correct. Only when the very last method is used, being really the last resort, are pencil and eraser required, since one might need to go back and erase some numbers.

Because practice makes perfect, we include a generous number of worked examples and exercises. These ingredients are essential for a good understanding of the techniques, as well as for mastering the given techniques. Last but not least the reader will find that our collection of techniques also works for the many variants of the traditional Sudoku that are now evolving, such as killer sudokus.

The author hopes that the reader will enjoy practicing the techniques in this book, and that in the end he will discover that he has become a real Sudoku expert!

Goirle, The Netherlands
April 2020
Wil Schilders

"I hope he has sudoku puzzles on him"

"I will be reading your chart as soon as I

## Rules of the game

The extreme popularity of sudoku can probably be explained by the fact that the rules of the game are explained in less than one minute, understandable for young and old. The starting point of each puzzle is a big square that is subdivided into 9 smaller main squares (often separated by boldfaced lines), that in turn are subdivided into 9 even smaller squares. Hence, we see 81 small squares of which a certain number are filled with numbers varying from 1 to 9 . The number of small squares in which a number is given at the start varies is often roughly between 20 and 40 . The diagram below shows an example.

|  | 1 |  |  |  | 2 | 9 |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  | 4 |  | 3 |  | 5 |
|  |  |  | 7 | 6 |  |  |  | 4 |
|  |  |  | 3 |  |  |  | 2 |  |
|  |  | 5 | 1 |  | 7 | 8 |  |  |
| 8 |  | 4 | 2 |  |  |  |  | 3 |
| 5 |  | 9 |  |  | 3 |  |  |  |
|  | 6 | 1 |  | 2 |  |  |  | 9 |
|  |  | 8 |  |  |  | 7 | 5 |  |

The intention is now to fill all 81 squares with numbers between 1 and 9 , in such a way that every row, every column and every main square contains the numbers 1 through 9 . Multiple occurrences of numbers are not allowed. Hence, the numbers 1 through 9 occur exactly once in every row, every column and each of the 9 main squares. In the diagram below, we see the result for the puzzle given on the foregoing page. Indeed, we see that all numbers occur only once in rows, columns and squares. All occurrences of the number 1 are shown in grey.

| 4 | 1 | 7 | 5 | 3 | 2 | 9 | 6 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 6 | 8 | 2 | 9 | 4 | 1 | 3 | 7 | 5 |
| 9 | 5 | 3 | 7 | 6 | 8 | 2 | 1 | 4 |
| 1 | 9 | 6 | 3 | 8 | 4 | 5 | 2 | 7 |
| 2 | 3 | 5 | 1 | 9 | 7 | 8 | 4 | 6 |
| 8 | 7 | 4 | 2 | 5 | 6 | 1 | 9 | 3 |
| 5 | 2 | 9 | 4 | 7 | 3 | 6 | 8 | 1 |
| 7 | 6 | 1 | 8 | 2 | 5 | 4 | 3 | 9 |
| 3 | 4 | 8 | 6 | 1 | 9 | 7 | 5 | 2 |

Hence, when one is solving a puzzle and finds that there are two 5 s in a row, this means that an error has been made. Thus, one needs to erase all numbers and start from the beginning or find the cause of the duplicate occurrence.

From the foregoing it is apparent that the rules of the game are very simple. This does not mean, however, that solving a sudoku puzzle is an easy task! This depends very much on the total number of squares filled at the beginning, and the exact position of these numbers. Some puzzles are completed easily, others take half an hour, and for some sudokus one will need several hours. In puzzle magazines this is often indicated with a classification using stars, 1 -star being easy, 4 -star being difficult, and 7 -star for extreme puzzles. However, this classification is not unique and not uniform, publishers use different classifications. It is rather difficult to present a good classification, and sometimes one will find that an "average" puzzle turns out to be extremely difficult to solve. For people starting to solve sudoku puzzles, it is advised to start with magazines classified as 1 -star or 2star, and slowly (and hopefully also steadily) increasing skills by trying more difficult puzzles. One will find that practice is quite important, just like in crosswords or cryptograms. Gradually, skills will grow!

## Exercises for forced Iocations

Finding forced locations for certain numbers turns out to be a very effective method to complete puzzles that appear hopeless at first sight. Of course, the method needs to be combined with techniques discussed before in Steps 1-4. Finding and using forced locations is not a simple task and it may cost a considerable amount of time before a new number to be filled in is determined. A systematic way of working is important in this context, as well as being alert to unusual situations and combinations. Therefore, we first present here a worked example to illustrate the power of the technique, followed by several puzzles that can all be solved using forced locations. First consider the following puzzle:

|  |  |  | 1 |  |  | 2 |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 3 |  |  |  |  |  | 4 |  |
|  |  |  |  | 5 |  |  |  |  |
|  | 5 |  |  |  | 3 |  |  |  |
|  |  |  | 7 |  |  |  |  |  |
| 8 |  |  |  | 9 |  |  |  |  |
| 7 |  | 2 |  |  |  |  |  |  |

Using the basic techniques in Steps 1 through 4 we can fill in quite a few empty cells. Here we give the numbers found, plus the steps used (using the numbering in the previous chapters) to find these numbers. This is done in the form of a table that should be read from left to right. Thus, every line contains three consecutive steps. The notation "ftg" refers to "filling the gap".

| Number | Position | Step | Number | Position | Step | Number | Position | Step |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7 | (R5,C3) | 1 | 5 | (R8,C3) | 2 | 5 | (R4,C6) | 3 |
| 5 | (R9,C4) | 1 | 2 | (R2,C5) | 3 | 7 | (R1,C5) | 3 |
| 7 | (R3,C2) | 1 | 2 | (R3,C1) | 1 | 3 | (R3,C4) | 2 |
| 7 | (R2,C7) | 1 | 2 | (R6,C2) | 1 | 3 | (R9,C5) | 1 |
| 7 | (R8,C8) | 1 | 7 | (R7,C6) | 1 | 2 | (R8,C6) | 1 |
| 1 | (R9,C6) | 4 | 8 | (R7,C4) | 4 | 6 | (R8,C4) | ftg |
| 2 | (R7,C9) | 1 | 2 | (R5,C8) | 1 | 4 | (R5,C4) | 3 |
| 9 | (R6,C6) | 4 | 9 | (R2,C4) | 1 | 5 | (R6,C7) | 2 |
| 4 | $(\mathrm{R6}, \mathrm{C} 9)$ | 2 |  |  |  |  |  |  |

Having arrived at this point, the diagram looks as follows:

|  |  |  | 1 | 7 |  | 2 |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 3 |  | 9 | 2 |  | 7 | 4 |  |
| 2 | 7 |  | 3 | 5 |  |  |  |  |
|  |  |  | 2 |  | 5 | 6 |  | 7 |
|  | 5 | 7 | 4 |  | 3 |  | 2 |  |
|  | 2 |  | 7 |  | 9 | 5 |  | 4 |
|  |  |  | 8 | 4 | 7 |  | 5 | 2 |
| 8 |  | 5 | 6 | 9 | 2 |  | 7 |  |
| 7 |  | 2 | 5 | 3 | 1 |  |  |  |

This looks promising: square $S 8$ is completely filled, and we have advanced the $4^{\text {th }}, 5^{\text {th }}$ and $6^{\text {th }}$ column quite a bit. Nevertheless, we find that the first 4 techniques do not lead to any new numbers. The ideal moment to check whether we can find any forced locations!

It takes some searching, but then we find that the number 3 in the $8^{\text {th }}$ row must be in square S 9 . Indeed, 3 is not allowed in ( $\mathrm{R} 8, \mathrm{C} 2$ ) due to the 3 in ( $\mathrm{R} 2, \mathrm{C} 2$ ) and the fact that the middle row in square S8 is already completely filled. Thus, we have drawn an ellipse around the middle row of square S 9 in the above diagram, indicating the forced location for the number 3.

## Non-traditional sudokus

Sudoku is very popular all over the world, with many people using it as a very pleasant pastime. This book shows that, even when one fully masters all techniques discussed, one still can have a lot of fun solving sudokus. But it is inevitable that variations to the traditional sudoku are being put onto the market. Hence, we see sudoku diagrams with additional grey main squares that must also contain the numbers 1 through 9 . Or puzzles for which the diagonals should contain all the different numbers. We see sudokus with letters or colours, we see 16 by 16 diagrams, twin sudokus with two combined diagrams requiring exchange of information, and so on.

For all these non-traditional sudoku puzzles, the techniques in this book are applicable. In order to demonstrate this, let's take an example:

| 3 | 9 | 5 | 4 |  |  | 7 | 8 |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 6 | 8 | 1 |  |  |  | 3 |  | 7 |  |
| 7 | 4 | 2 | 5 |  |  |  |  |  | 6 |
|  |  |  |  |  |  |  |  | 8 |  |
|  | 3 |  | 8 |  |  | 5 |  |  |  |
|  |  | 6 | 3 |  | 7 | 2 |  | 1 | 4 |
|  |  | 8 |  |  |  |  | 1 | 5 | 9 |
| 4 |  |  |  |  |  | 6 | 2 | 3 | 8 |
| 5 |  | 3 |  |  | 2 |  | 6 | 4 | 7 |

The grey areas should contain all numbers 1 through to 9 . If the reader has been successful, he will find the following solution, from which we can clearly see that indeed the grey areas have the required property.

| 3 | 9 | 5 | 4 | 6 | 7 | 8 | 2 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 6 | 8 | 1 | 2 | 9 | 3 | 4 | 7 | 5 |
| 7 | 4 | 2 | 5 | 8 | 1 | 3 | 9 | 6 |
| 1 | 2 | 7 | 6 | 4 | 9 | 5 | 8 | 3 |
| 9 | 3 | 4 | 8 | 1 | 5 | 7 | 6 | 2 |
| 8 | 5 | 6 | 3 | 7 | 2 | 9 | 1 | 4 |
| 2 | 6 | 8 | 7 | 3 | 4 | 1 | 5 | 9 |
| 4 | 7 | 9 | 1 | 5 | 6 | 2 | 3 | 8 |
| 5 | 1 | 3 | 9 | 2 | 8 | 6 | 4 | 7 |

We now provide some sudokus for which the reader can practice his skills. In the first of these, the two diagonals also need to contain the numbers 1 through 9 , whereas in the second one the nine grey cells should contain the numbers 1-9. The last two puzzles are more advanced ${ }^{3}$. These sudokus contain 4 additional main squares (in grey), which should also contain the numbers 1 through 9 . Although our techniques also apply to these puzzles, it takes some time to get used to the slightly different strategies. And, of course, it can be a bit harder as one now needs to extract information from 13 main squares rather than 9 . Curious about these sudokus is that when one tries to attempt to solve these using computer programs on the internet, such as the one at http://www.sudokuhints.com, one will notice that the program will tell you that it is stuck at some point. This is not surprising: it will treat the diagram as a normal sudoku, and the challenge is to extract extra information from the fact that the grey cells must also be filled with 1-9.

[^2]
[^0]:    ${ }^{1}$ Some claim that the first sudoku puzzle appeared in the American magazine "Dell Puzzle Magazine" in 1979 and was used by a Japanese publisher in 1984. Others see a resemblance to even earlier publications.

[^1]:    ${ }^{2}$ There is a nice booklet by Robin Wilson entitled "How to solve sudoku", published by Sterling in New York. It is easily accessible and has detailed explanations.

[^2]:    ${ }^{3}$ These puzzles are an invention of Peter Ritmeester and published in the Dutch newspaper NRC, but also in The International Herald tribune and the China Daily.

