ELEGANCE WITH SUBSTANCE

# Elegance with Substance

Mathematics and its education designed for Ladies and Gentlemen

What is wrong with mathematics education and how it can be righted

On the political economy of mathematics and its education

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# Prologue to the 1st edition in 2009

Our pupils and students are best treated as ladies and gentlemen with elegance and substance. Providing them with equal mathematics is our much valued objective.

Ideally mathematics would be perfect and unchanging and just be there to be discovered. Mathematics however is as much art as discovery. It is made. It is a creation, in the way that cavemen carved their scores in bones and that we create virtual reality with supercomputers. In the interaction between what we do and what we understand almost all of the weight is on what we do, which then imprints on our mind. It appears tedious and hard work to go a bit in the reverse direction, to even get where we are now, let alone develop a notion of perfection.

Given this fragile and historic nature of mathematics it should not come as a surprise that what we currently call mathematics actually appears, on close inspection, to be often cumbersome or even outright irrational. Clarity and understanding are frequently blocked by contradictions and nonsense that are internal to current mathematics itself. Who has a problem mastering mathematics should not be surprised.

Over the years, while teaching mathematics and writing my notes that now result in these pages, there were many moments that I felt frustrated and at times even quite annoyed about the straightjacket of current mathematical conventions. One is supposed to teach mathematics but it is precisely the textbook that blocks this prospect. For many pupils and students the goal is impossible from the outset not because of their lack of capability but because of awkward conventions that only came about in a historical process.

The flip side is that this is a Garden of Eden for didactic development. What is awkward can be hammered into something elegant. What is irrational can be turned rational and consistent. What is dark and nonsensical can be thrown out and replaced by clarity. There is beauty and satisfaction in redesign.

This didactic reconsideration also changes what we call ‘mathematics’. The interaction between what we do and what we understand shifts to a new equilibrium, a higher optimum at a more agreeable level for both students and teacher. It will still be mathematics since it can be recognized as mathematics. It will be stronger and more efficient mathematics too but it will no longer be the old one.

The criterion for change lies in elegance with substance. Elegance without substance creates a dandy. Elegance ought to signal substance. Mathematics concentrates on the elegance and specific fields of study like economics concentrate on the substance. But mathematics needs to have some substance of itself too. The criterion is tricky since some people see it in the present mathematical conventions too, where awkwardness $A$ plus awkwardness $B$ gives inconsistency $C$. However, we will compare the old ways with the suggestions of the new ways and let the criterion speak for itself. This should open some eyes. Otherwise we just stay in the Garden of Eden.

Which leaves me to thank my own teachers and colleagues who trained and helped me in the old ways. A redesign starts from something and when the old is replaced then this implies that it was valuable to start with. I thank in particular my pupils and students for what they taught me.

# Prologue to the 2nd edition in 2015

This 2nd edition is almost the same as the 1st edition. *Elegance with Substance* (EWS) has a strong structure, a wealth of information, and not too many pages. It argues its case succinctly. This clarity will be lost when new issues are included.

The results from 2009 generally haven't changed. When developments in 2009-2015 supplement some of the results then a footnote explains where more can be found.

Major new developments in 2009-2015 have been:

- *Conquest of the Plane* (COTP) (2011c) presents a *proof of concept* for EWS and creates a *primer* for a re-engineered course in mathematics for highschool or first year of higher education. Gill (2012) reviewed both EWS and COTP favourably and advised to read with an open mind. At the review site of the European Mathematical Society, Gamboa (2011) states about COTP: "Once the reader becomes familiar with the notations and the author style, he/she will enjoy the book. I am convinced that this work will help the students to recognize what they should know but they ignore, (...)"
- *Foundations of Mathematics. A Neoclassical Approach to Infinity* (FMNAI) (2015f) argues the case for having set theory and number theory in the highschool programme. A major deduction is that the transfinites by Georg Cantor are based upon ill-defined constructs, comparable to Russell's paradox, so that these can be eliminated. As a result the theory for the classroom is not as complex as commonly perceived.

The memo *What a mathematician might wish to know about my work* (2013) has been included on page 125 in the Appendices. The *Introduction* already explains about my background in econometrics and teaching mathematics, but this memo emphasizes again that my interest is not abstraction for the purpose of abstraction itself, but to create scope for betterment of society. Obviously, society will not be improved when an applied theory is unsound, and thus there is a role for mathematics, but here the sounding board is not only abstraction itself but also empirical science.

# Contents in brief

# Contents

# I. Introduction

## 1. Natural limitations to a noble art

A distinction that comes natural to us is between empirical reality and abstract thought. The first is the subject of the empirical sciences, the latter the realm for mathematics and ideal philosophy. This distinction comes with the observation that mathematicians are little trained in empirical issues.

Our subject is the education in mathematics.

Didactics, and in particular the didactics of mathematics, deals with real pupils and students. Didactics requires a mindset that is sensitive to empirical observation – which is not what mathematicians are trained for.

## 2. As far as the mind can reach

Mathematics is a great liberating force. No dictator forces you to accept the truth of the Pythagorean Theorem. You are free to check it for yourself. You may even object to its assumptions and invent non-Euclidean geometry. Mathematical reasoning is all about ideas and deductions and about how far your free mind will get you – which is amazingly far.

But you have to be aware of reality if you say something about reality.

The education in mathematics is not without some empirical study. It is proper to recall the Van Streun (2006) *In Memoriam* of A.D. de Groot. It is a painful point however that such exceptions prove the rule.

### ***For the record***

The stock market crash in Autumn 2008 caused criticism on mathematicians and the 'rocket scientists' by Mandelbrot & Taleb (2009), Taleb (2009) and Salmon (2009). The mathematicians involved overlooked the difference between their models and reality. Accents differ a bit: Mandelbrot more on other solutions on the assumptions on the law of large numbers, Taleb more on risk, Salmon more on correlation. It remains amazing that the mathematicians at the banks and hedge funds did not issue a warning somewhere in the processs and it would be obvious that those cannot evade part of the responsibility. Of course, there is a lot of blame to go around. Like the rest of the world, Taleb (2009) and Salmon (2009) are also critical on economists and lawyers in bank management and financial regulation. Fortunately, I am one those economists who issued a warning.

With respect to ecological collapse, Tinbergen & Hueting (1991) presented an approach to monitor how the economy affects the environment and to keep account of ecological survival. Their economic approach pays attention to statistics and real risks as indicated by ecologists. Alternatives came notably from modellers with a mathematical mindset who put emphasis on elegant form and easy notions of risk. Those models suggest that there are no relevant risks on the ecology, which is an agreeable suggestion for most policy makers. Final responsibility falls on those policy makers and society who allow this to happen but it remains strange that those modellers think that they contribute more than only their own assumptions and deductions. See THAEES, Colignatus (2009, 2015).

With respect to logic and democracy, Colignatus (2007ab, 2008b), updated from 1981 / 1990, considers statements by mathematicians that have been accepted throughout academia and subsequently society on the basis of mathematical authority. It appears however that those statements mix up true mathematical results with interpretations about reality. When these interpretations are modelled mathematically, the statements reduce to falsehoods. Society has been awfully off-track on basic notions of logic, civic discourse and democracy. Even in 2007, mathematicians working on voting theory wrote a *Letter to the governments of the EU member states* advising the use of the Penrose Square Root Weights (PSRW) for the EU Council of Ministers. See Colignatus (2007c) on their statistical inadequacy and their misrepresentation of both morality and reality.

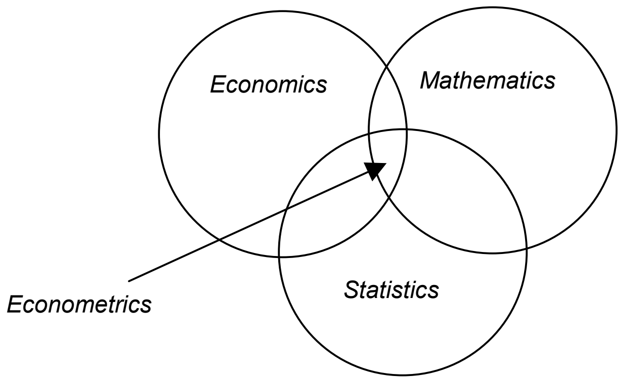
Over the millenia a tradition and culture of mathematics has grown that conditions mathematicians to, well, what mathematicians do. Which is not empirical analysis. Psychology will play a role too in the filtering out of those students who will later become mathematicians. After graduation, mathematicians either have a tenure track in (pure) mathematics or they are absorbed into other fields such as physics, economics of psychology. They tend to take along their basic training in abstraction and then try to become empirical scientists – but within the framework of their basic training.

The result is comparable to what happens when mathematicians become educators in mathematics. They succeed easily in replicating the conditioning and in the filtering out of new recruits who adapt to the treatment. For other pupils it is hard pounding.

## *Definition of econometrics*

My own training in mathematics, as a student of econometrics starting Autumn 1973, was with the students of mathematics, physics and astronomy. Thus I do not feel any shortcomings here. My mathematical results e.g. in Colignatus (2007ab) are quite nice even though not developed axiomatically. I have limited affection for pure mathematics but am aware of the hesitations on their part. At least I have the training not to claim more than can be proven, to distinguish fact and hypothesis. For me, however, this holds both in mathematics and about reality. For readers not familiar with the notion of econometrics, I can usefully replicate the diagram by Rijken van Olst, see **Figure 1**.

**Figure 1. The Rijken van Olst diagram for econometrics**




Some see econometrics as a specialisation but actually it is a generalisation that allows one to work on all angles. Specialists in one of the angles can get deeper results and generalisation comes with modesty, but this generalisation is the only way if we want to tackle reality in scientific fashion.